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Stochastic Programming Model for Lateral Transshipment Considering Rentals and Returns

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Abstract

Supply chain management is a large scale planning under uncertainty. It is significant that building an efficient supply chain under uncertain condition; however, it is difficult. There are many traditional inventory transshipment models targeting only rental, not including return. This study provides the uncertainty between rentals and returns by scenarios with multi-period stochastic programming model of inventory transshipment problems. The moment matching method was used to reduce the amount of the scenarios, and the comparative experiment shows the utility of this study model.

Keywords: stochastic programming, inventory transshipment, supply chain, moment matching method

1 Introduction

In recent years, it has become a difficult problem to minimize inventory costs, ordering costs, and shipping costs while satisfying the various needs of each customers. There are much research of facility location problem and inventory planning problem by stochastic programming to build an efficient supply chain under uncertain condition. In addition, supply chain management in multi-period involves a long time to calculate. Therefore, many studies using K-means and moment matching method, which reduce the number of scenarios, has been performed in order to make realizable time to calculate.

Paterson et al. [4] reviewed and categorised previous studies of lateral transshipment. Lateral transshipment models are categorised into two categories: one of them occurs at predetermined time before all demand is known and another one can be done at any time to accomodate a potential shortage.

Shiina et al. [7] proposed the inventory transshipment problem that aims to enhance the service level and minimize the costs by cutting down on the numbers of orders to factories by inventory transshipment between the bases in consideration of demand fluctuation. In this study, stochastic programming is used to consider the penalty for causing inventory shortages. However, the model is a single period, it can be extended to the multi-period model.

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The research of Schildbach and Morari [5] aims to minimize production cost, shipping cost and inventory cost in multi-period supply chain management while taking demand fluctuation into consideration. Their research makes a decision to meet demand invariably without consideration of the inventory shortage.

Aragane et al. [1] suggests the inventory transshipment problem to minimize the total cost by using scenario tree to provide the demand fluctuation in multi period considering rentals and returns. In their research, there is a rule to set up a new factory when it is difficult to meet demand with the present condition. The number of scenarios is reduced by using the K-means method, which aggregates scenario fans into a scenario tree.

2 Stochastic Programming Problem

In this study, the inventory transshipment problem is formulated as a two-stage model with recourse using stochastic programming. We form the basic two-stage stochastic linear programming problem with recourse as (SPR).

(SPR)

min

$$c^\top x + \mathscr{Q}(x)$$

s.t.

$$Ax = b, x \geq 0$$

$$\mathscr{Q}(x) = E_{\tilde{\xi}}[Q(x, \tilde{\xi})]$$

$$Q(x,\xi) = \min\{q(\xi)^\top y(\xi) \mid Wy(\xi) = h(\xi) - T(\xi)x, y(\xi) \geq 0\}, \xi \in \Xi$$

In the formulation of (SPR), c is a known n_1-vector, b is a known m_1-vector, $q(>0)$ is a known n_2-vector, and A and W are known matrices of size $m_1 \times n_1$ and $m_2 \times n_2$, respectively. The first stage decision are represented by the n_1-vector x. We assume the m_2-random vector $\tilde{\xi}$ is defined on a known probablity space. Let Ξ be the suppor of $\tilde{\xi}$, that is, rfc qk_jjcqr closed set for which P(Ξ)=1 holds. Given a first stage decision x, the realization of random vector ξ of $\tilde{\xi}$ is observed. The second stage data ξ become known. Then, the second stage decision $y(\xi)$ must be taken so as to satisfy the constraints $Wy(\xi) \geq \xi - Tx$ and $y(\xi) \geq 0$. The second stage decision $y(\xi)$ is assumed to cause a penalty of q. The objective function contains a deterministic term $c^\top x$ and the expectation of the second stage objective. The symbol $E_{\tilde{\xi}}$ represents the mathematical expectation with respect to $\tilde{\xi}$, and the function $Q(x,\xi)$ is called the recourse function in stage ξ. The value of the recourse function is given by solving a second stage linear programming problem.

3 Multi-Period Inventory Transshipment Problem

3.1 Problem description

This study aims to minimize the total cost of a multi-period inventory transshipment problem. This study is considered for businesses to deal with rental products such as large container houses. Aragane et al. [1] divided the network into three echelons: the first echelon is the factory, the second echelon is the base, and the third echelon is the customer. This model assumes that customer demand and the length of the rental period follow a probability distribution. In typical supply chain, a large amount of fixed order costs is incurred when each base places an order with a factory every period. Thus, transshipment between bases, managing a risk of inventory shortage by inventory transshipment between bases, improves the service level while reducing the amount of ordering to a factory. This study uses inventory transshipment without distinction of it between before and after proving the demand. Each base performs inventory transshipment from a base which is overstocked or has cheap order costs to other bases which has inventory shortage or has expensive order costs, at the same time that each base orders to a factory. Fig.1 shows an overview of the inventory transsipment problem considering rentals and returns. The preconditions for this study are as follows.

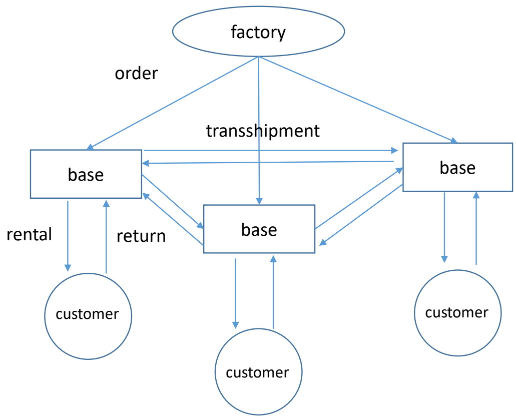

Figure 1: Overview of the model

- Each base may rent a product only once, and the rental product will be returned to the same base in some period.
- The rental is made before the final period and the return is made in the period after the rental.
- Each base keeps products by orders from factories and inventory transshipment.
- The rental and the return are made at the same base.
- Fixed order costs are incurred when placing orders for factories.
- Penalty are incurred for inventory shortages caused by uncertain demand.

By constructing a scenario tree, we represent uncertainty in multiple periods. The scenario tree consists of a single root node at the initial stage and branches to a finite number of descendant nodes at each stage. The scenario tree has the characteristics of branching structure and non-anticipativity. Non-anticipativity means that the decision maker can not make a decision based on the anticipation that the decision branches into different scenarios in the future. The typical scenario tree when the number of branches is set to 2 is shown in Fig2.

If the period is T and the number of bases is I, the number of rental and return patterns for each location can be expressed as $\frac{T(T-1)}{2}$. In this study, the number of rental and return patterns at each base is determined by the period, so the total number of scenarios depends

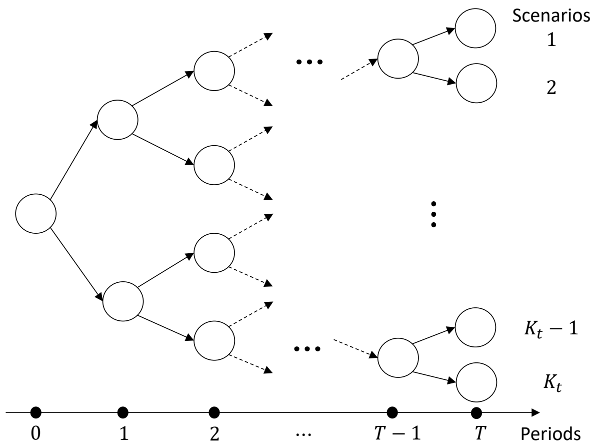

Figure 2: Scenario tree

on the number of bases and the period. The total number of scenarios can be obtained by multiplying the number of rental and return patterns by the number of bases. Fig.3 shows the number of rental and return patterns for the period 4. Blue circle shows the period when rental occurs, and red circle shows the period when to be returned. White circle shows the period when neither occurred.

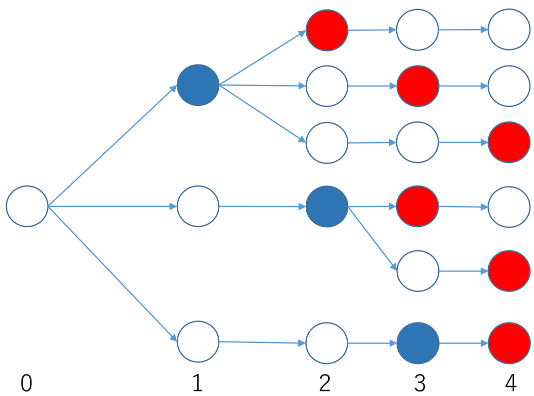

Figure 3: Patterns of the model

In addition, we consider the probability of each of the rental and return patterns in this study. By defining probabilities for rental and return, we set some probabilities to all rental and return patterns. In each period, rental is assumed to occur in period n with equal probability $\frac{1}{T-1}$. Return is also assumed to occur with equal probability $\frac{1}{T-n}$ in each period. The probability of the rental and return pattern for period 4 is calculated as follows.

- Probability of scenarios 1,2,3 $\frac{1}{(4-1)(4-1)} = \frac{1}{9}$
- Probability of scenarios 4,5 $\frac{1}{(4-1)(4-2)} = \frac{1}{6}$
- Probability of scenarios 6 $\frac{1}{(4-1)(4-3)} = \frac{1}{3}$

Since each scenario is determined by the period when the rental occurred, the probability of the scenario can be shown as $\frac{1}{(T-1)(T-n)}$ if the period when the rental occurred is n. The probability of the rental and return pattern for period 4 can be shown in Fig. 4.

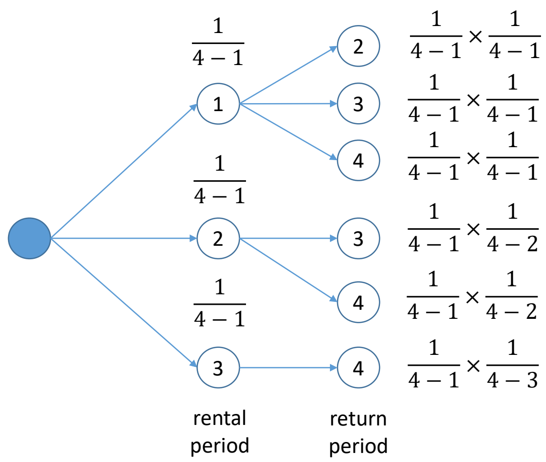

Figure 4: Probability of patterns in the model

3.2 Notations

Sets

I	The set of bases
S	The set of scenarios
T	The set of periods

Parameters

ξ_{it}^s	Rental quantity to base i under scenario s in period t
ζ_{it}^s	Return quantity from base i under scenario s in period t
H_{it}^s	Inventory cost of base i under scenario s in period t
P_{it}^s	Stockout cost of base i in scenario s of period t
R_{it}^s	Variable order cost of base i under scenario s in period t
L_{ijt}^s	Lateral transshipment cost from base i to base j under scenario s in period t
W_{it}	Fixed order cost of base i in period t
β	Reusable rate of returned products
CAP	Capacity of the factory
p^s	Occurrence probability of scenario s

Variables

u_{it}	0-1 decision variable, 1 if the order is placed at base i in period t, otherwise 0
I_{it}^s	Inventory quantity of base i under scenario s in period t
z_{it}^s	Stockout quantity of base i under scenario s in period t
o_{it}^s	Order quantity of the base i to factory under scenario s in period t
x_{ijt}^s	Lateral transshipment quantity from base i to base j under scenario s in period t

The 0-1 variables related to the order decision were defined as scenario-independent decision variables, and the other decision variables were defined for each scenario.

3.3 Formulation

The proposed problem is formulated as a multi-period stochastic problem with scenarios. The problem can be formulated as follows.

$$\min \sum_{t\in T}\sum_{i\in I} W_{it}u_{it} + \sum_{t\in T}\sum_{s\in S} p^s(\sum_{i\in I}\sum_{j\neq i} L^s_{ijt}x^s_{ijt} + \sum_{i\in I} R^s_{it}o^s_{it} + \sum_{i\in I} H^s_{it}l^s_{it} + \sum_{i\in I} P^s_{it}z^s_{it}) \tag{1}$$

s.t.

$$\sum_{i\in I} o^s_{it} \leq CAP \qquad \forall t\in T, \forall s\in S \tag{2}$$

$$l^s_{it} - z^s_{it} = l^s_{it-1} + o^s_{it} + \sum_{j\in I} x^s_{jit} - \sum_{j\in I} x^s_{ijt}$$

$$-\xi^s_{it} + \beta\zeta^s_{it}, \quad \forall i\in I, \forall t\in T, \forall s\in S \tag{3}$$

$$\sum_{j\in I} x^s_{ijt} \leq l^s_{it-1} + o^s_{it}, \forall i\in I, \forall t\in T, \forall s\in S \tag{4}$$

$$o^s_{it} \leq Mu_{it}, \qquad \forall i\in I, \forall t\in T, \forall s\in S \tag{5}$$

$$u_{it}\in\{0,1\}, l^s_{it}, z^s_{it}, o^s_{it}, x^s_{ijt} \geq 0$$

$$\forall i\in I, \forall t\in T, \forall s\in S \tag{6}$$

The objective function (1) of the main problem represents the minimization of the total cost that is the sum of ordering fixed cost, inventory flexibility cost, ordering cost, inventory cost, and inventory shortage cost. Inequality (2) is the constraint that the amount of orders placed with the factory is less than or equal to the capacity of the factory. Inequality (3) is the constraint representing the conservation of the amount of inventory at the base. The left side of (3) represents the amount of inventory in period t of scenario s, and the right side represents the sum of the amount of inventory in period $t-1$ of scenario s and the transition in products at base i in period t. The amount of products are changed due to orders, rentals, returns, and inventory transshipment between bases. Inequality (4) is a constraint that the amount of inventory transshipment must be less than or equal to the amount of inventory at the end of the previous period, and inequality (5) is a constraint that represents the upper bound on the amount of orders.

4 Moment Matching Method

4.1 Overview of the moment matching method

The total number of scenarios in this study increases with the number of bases and the period. When the total number of scenarios is huge, the computation time also becomes very long. In this study, we try to reduce the number of scenarios and the computation time by using the moment matching method. Kaut and Wallace [9] presented an overview of the most basic scenario generation methods. They classified these methods into methods for generate scenario trees and methods for generate scenario fans. If the uncertainty is represented by a multivariate continuous distribution or a discrete distribution with too many outcomes, there is a need to reduce the number of possible scenarios. Aragane et al. [1] used K-means method to solve the inventory transshipment problem. This method aggregates the scenario fans which has a large number of scenarios and large scale of the problem into a scenario tree with a smaller number of nodes by bundling the nodes of each period into clusters. Hoyland and Wallace [2] deal with the moment matching method that reduces scenarios by approximate solutions without drastically changing the nature of the distribution by giving the probabilities again. If the scenario is given in the form of a tree, this method sets the probability of the scenario for each stage again, keeping the statistical properties such as the expected value, variance, skewness, and kurtosis in the original probability distribution. If the probability is given once again, the probability of the node corresponding to each stage scenario may become zero, then the probability of all the child nodes of that node will also become zero. Therefore, it is possible to reduce the number of scenarios.

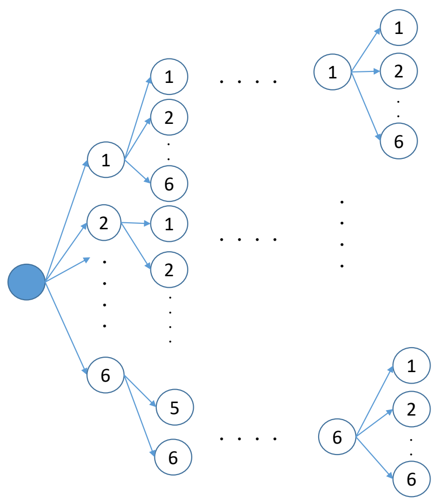

Figure 5: Scenario tree for 5 bases

4.2 Notation of the moment matching method

Parameters

N	Node set where probability fluctuation occurs
b_i	The number of fluctuations at base i
B_i	Total number of nodes in the scenario tree corresponding to base i
$\overline{M}_i, \overline{Q}_i$	Expected value of the period when rental occurs or to be returned at base i
Σ_i, C_i, V_i	Variance, skewness, and kurtosis of the period when rental occurs at base i
σ_i, c_i, v_i	Variance, skewness, and kurtosis of the period when to be returned at base i
X_i^k, Y_i^k	The kth realization in the scenario tree for the period when rental occurs or to be returned at base i
λ_i	Weight factor at base i
$\omega_i^1, \omega_i^2, \omega_i^3$ $\mu_i^1, \mu_i^2, \mu_i^3$	Weight factor for variance, skewness, and kurtosis of the period when rental occurs or to be returned at base i

Variables

p_i^k	Probability of occurrence of the kth realization in the scenario tree at base i
Σ_i^+, Σ_i^-	Difference from the theoretical value of the variance of the period when rental occurs at base i
σ_i^+, σ_i^-	Difference from the theoretical value of the variance of the period when to be returned at base i
C_i^+, C_i^-	Difference from the theoretical value of the skewness of the period when rental occurs at base i
c_i^+, c_i^-	Difference from the theoretical value of the skewness of the period when to be returned at base i
V_i^+, V_i^-	Difference from the theoretical value of the kurtosis of the period when rental occurs at base i
v_i^+, v_i^-	Difference from the theoretical value of the kurtosis of the period when to be returned at base i

4.3 Formulation of the moment matching method

The moment matching method is formulated as follows.

$$\min \sum_{i=1}^{N} \lambda_i(\omega_i^1(\Sigma_i^+ + \Sigma_i^-) + \omega_i^2(C_i^+ + C_i^-) + \omega_i^3(V_i^+ + V_i^-)$$

$$+ \mu_i^1(\sigma_i^+ + \sigma_i^-) + \mu_i^2(c_i^+ + c_i^-) + \mu_i^3(v_i^+ + v_i^-)) \tag{7}$$

s.t.

$$\sum_{k=1}^{B_i} X_i^k p_i^k = \overline{M}_i, i = 1..N \tag{8}$$

$$\sum_{k=1}^{B_i} Y_i^k p_i^k = \overline{Q}_i, i = 1..N \tag{9}$$

$$\sum_{k=(j-1)b_i+1}^{jb_i} p_i^k = p_{i-1}^j, i = 1..N, j = 1..B_{i-1} \tag{10}$$

$$\sum_{k=1}^{B_i} (X_i^k - \overline{M}_i)^2 p_i^k - \Sigma_i^+ + \Sigma_i^- = \Sigma_i, i = 1..N \tag{11}$$

$$\sum_{k=1}^{B_i} (Y_i^k - \overline{Q}_i)^2 p_i^k - \sigma_i^+ + \sigma_i^- = \sigma_i, i = 1..N \tag{12}$$

$$\sum_{k=1}^{B_i} (X_i^k - \overline{M}_i)^3 p_i^k - C_i^+ + C_i^- = C_i, i = 1..N \tag{13}$$

$$\sum_{k=1}^{B_i} (Y_i^k - \overline{Q}_i)^3 p_i^k - c_i^+ + c_i^- = c_i, i = 1..N \tag{14}$$

$$\sum_{k=1}^{B_i} (X_i^k - \overline{M}_i)^4 p_i^k - V_i^+ + V_i^- = V_i, i = 1..N \tag{15}$$

$$\sum_{k=1}^{B_i} (Y_i^k - \overline{Q}_i)^4 p_i^k - v_i^+ + v_i^- = v_i, i = 1..N \tag{16}$$

$$\Sigma_i^+, \Sigma_i^-, \sigma_i^+, \sigma_i^-, C_i^+, C_i^-, c_i^+, c_i^-, V_i^+, V_i^-, v_i^+, v_i^- \geq 0 \tag{17}$$

The objective function (7) minimizes the difference between the theoretical value and approximated value of the variance, skewness, and kurtosis. Equation (8) (9) are constraints which are the product of the realization and the probability is equal to the theoretical value of the expectation. Equation (10) is the constraint that the sum of the descendant probabilities in the scenario tree is equal to the probability of the parent node. Equation (11) and (12) give the variance, Equation (13) and (14) give the skewness, and Equation (15) and (16) give the difference from the expected value of kurtosis. The problem became a linear programming problem because the coefficients of the random variable p_i^k for Equation (11) to (16) are constant by deciding as Equation (8) and (9).

5 Numerical Experiments

The locations of factory and bases are randomly generated on [0,100]×[0,100] grid. The order cost R_{it}^s is defined as (1+0.1×the distance between the factory and the base. The lateral transshipment cost L_{ijt}^s is defined as (0.5×the distance between the bases). The fluctuation of rentals and returns was represented by a scenario tree. The rental quantity follows a normal distribution with a standard deviation of 10. The mean of base1, 2, 3, 4, and 5 are 100, 150, 200, 250, and 300, respectively. The number of scenarios varies depending on the period and the number of bases, and is represented by a scenario tree. Values of major parameters are described in following Table 1.

Table 1: Values of major parameters

Notation		Value
The number of bases	I	5
Inventory cost	H_{it}^s	1
Stockout cost	P_{it}^s	10
Order fixed cost	W_{it}	50
Reusable rate of returned products	β	0.25
Capacity of the factory	CAP	2000

The purpose of the numerical experiments is to demonstrate the effectiveness of the proposed model, moreover to reduce the scenario and computation time by using the moment matching method. First, we show the usefulness of this model by comparing it with a deterministic problem. Next, we compare the computation time of the problem applying the moment matching method with the original problem.

In evaluating the solution of a stochastic programming method, the value of the stochastic solution (VSS) is used. The deterministic model is formulated by replacing the random variable with the mean of all scenarios. Let EEV be the optimal objective value of the stochastic programming problem with the optimal solution of the deterministic problem. Let RP be the optimal objective value of the stochastic programming problem. VSS is defined using EEV and RP as follows.

$$\text{VSS} = \text{EEV} - \text{RP} \tag{18}$$

Table 2 shows the results of an experiment comparing RP and EEV. Improvement rate of a stochastic model is defined as follows.

$$\text{improvement rate}[\%] = \frac{VSS}{EEV} \times 100 \tag{19}$$

In all cases, RP gives a better solution than EEV. The improvement rate in the case of 4 bases was the highest in the range of numerical experiments. For any number of locations, the number of orders placed was lower for EEVs, but the quantity ordered at one time was larger than for RPs. The effectiveness of a stochastic programming model is shown in Table 2.

Table 2: Evaluation of solution for stochastic programming model

The number of bases	RP	EEV	VSS	Improvement rate[%]
4	3807.2	4256.3	449.1	10.6
5	5437.0	5897.4	460.4	7.81
6	7729.6	8610.9	881.3	10.2

The results of the computation time and the number of scenarios in the Table 3 and 4 show the effectiveness of the moment matching method. In this experiment, the period was set to 4 and experiments were conducted for the cases of 5 or 6 bases. The probabilities of each scenario were recomputed from the newly obtained probabilities by the problem applying the moment matching method. Using these probabilities, we solved the original problem. In both cases, we were able to significantly reduce the number of scenarios and computation time. The total number of scenarios was reduced more when the number of bases was 6 than when the number of bases was 5. This shows that the number of scenarios can be reduced as the number of bases increases.

For the original problem, the error rate of the objective function is kept within 2%. It indicates that the moment matching method is working effectively. Fig. 6 and Fig. 7 show the scenario tree applying the moment matching method for the cases of 5 and 6 bases.

Table 3: Experimental results in the case of 5 bases

	Total cost	Computation times	Number of scenarios	Error[%]
Original problem	5437.0	413.1	7776	-
Moment matching	5546.8	1.4	19	-2.021

Table 4: Experimental results in the case of 6 bases

	Total cost	Computation times	Number of scenarios	Error[%]
Original problem	7729.6	8768.2	46656	-
Moment matching	7649.3	12.4	25	1.039

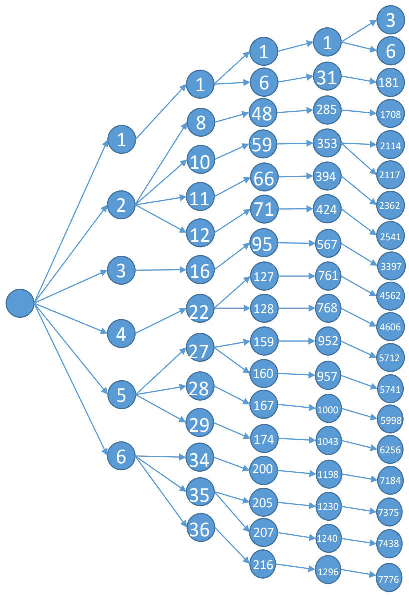

Figure 6: Scenario tree of the problem applying moment matching in the case of 5 bases

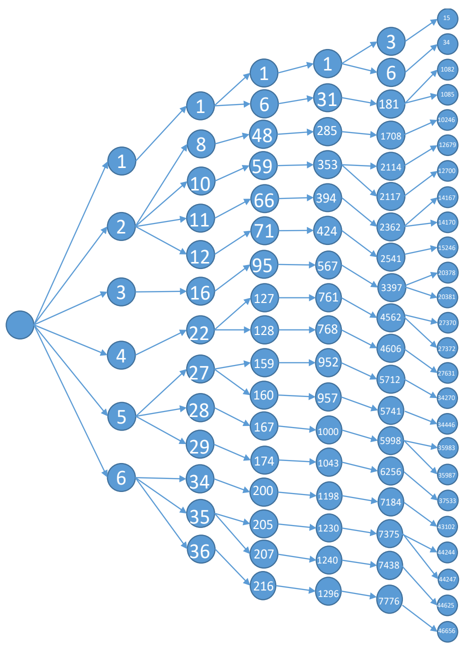

Figure 7: Scenario tree of the problem applying moment matching in the case of 6 bases

6 Conclusion

In this study, the inventory transshipment problem is extended to a multi-period stochastic programming problem considering rentals and returns. The results of numerical experiments demonstrate the effectiveness of the stochastic model. Moreover, the computation time and the number of scenarios were reduced by using the moment matching method. The error of total cost can be suppressed to a small value within 2%.

As a future issue is a multi-period planning that takes into account the lead time for ordering. This study shows the results by only changing the number of locations; however, further application of the problem is necessary to provide the effectiveness of the method.

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