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Investigation of Deterministic Particle Swarm Optimization with Periodic Function

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Abstract

This study proposes a deterministic particle swarm optimization method that introduces a periodic function (DPSOP). Particle swarm optimization (PSO) is an evolutionary algorithm. The agents have positions and velocities that are updated according to the global and local best solutions. The moving length of agent is generally randomly determined. In the proposed DPSOP method, the moving length of agent is determined by trigonometric function. Each agent has different phases, and the moving length of agent to the global and local best solutions changes according to the cosine and sine waves, respectively. Moving for the global best solution is out of phase with the local best by $\frac{\pi}{2}$. Depending on the phase difference, the agent's movement is divided into a time period when it is maximally near the global and local best solutions. Therefore, DPSOP can perform an efficient solution search. We confirm the performance of the proposed DPSOP method using the search solutions of five benchmark functions.

Keywords: Particle swarm optimization, sine-cosine wave, deterministic, optimization.

1 Introduction

Currently, many types of artificial intelligences (AI) have proposed. In the AI, evolutionary computation algorithms obtains constant evaluation because it does not need derivative for optimization, such as the genetic algorithm (GA), firefly algorithm (FA), and particle swarm optimization (PSO) [1]–[5]. They are characterized by their ability to perform optimization without differentiation and can be adapted to various optimization problems. Among them, PSO is a well-known swarm intelligence method based on moving swarms of fish or birds [5][6]. PSO does not use the derivative in searching for the solution and converges rapidly. Therefore, many researchers have proposed improving PSO and have applied it to various optimization problems [7]–[11]. PSO is constructed using many agents that have a position (x) and velocity (v). The position is an optimization target with high dimensions, according to the task. The velocity is a moving vector in the solution space. The position of each agent is updated based on the current velocity. The current velocity is calculated using the momentum of the previous velocity, the distance between the current position and global best (x_{gbest}), and the distance between the current position and local best (x_{pbest}). The global

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best is the best solution found by all agents before the current iteration. The local best is the best solution obtained by each agent before the current iteration. In standard PSO, the length of the update vector is randomly determined; therefore, standard PSO is a stochastic search solution. Moreover, some researchers have proposed deterministic PSO. Deterministic PSO does not use a random vector; thus, the convergence of deterministic PSO is faster, and the search result is more stable than that of standard PSO [12]–[16]. However, deterministic PSO often falls into a local minimum because the solution search depends only on the deterministic value.

This study presents a deterministic PSO method based on periodic function (DPSOP). DPSOP is updated by the velocity and position according to the global and local best solutions same as standard PSO. In DPSOP, the length of the update vector that is near the global and local best is determined by cosine and sine waves. We provide cosine and sine waves to move near the global and local best solutions, respectively. Therefore, the length of moving near the global and local best solutions has a quadrature phase. In addition, each agent has a different initial phase, which is shifted at the same speed between agents with iterations. As mentioned, the agents approach only the global or local best within a certain period in the proposed DPSOP. The moving distance of the original PSO approach is determined at random with each iteration. In DPSOP, the moving distance changes smoothly with each iteration. Therefore, DPSOP particles can be expected to continue moving while maintaining their velocity, which is an advantage of DPSOP over the original PSO approach. We believe that the orthogonal phase between approaching the global and local best improves the solution searching ability of PSO. We compared the proposed DPSOP with the standard PSO using five benchmark functions, including unimodal and multimodal functions.

2 Proposed Method

In this section, we explain the PSO algorithm and proposed DPSOP. PSO is a well-known swarm-intelligence method. This algorithm has been applied to various optimization tasks, because it does not use the derivatives of the target function for optimization. In general, the position of standard PSO updates according to the x_{gbest} and x_{pbest}, and these updating distances are determined by random values. We introduce the following periodic function.

2.1 PSO

The standard PSO algorithm is as follows.

(1) The agents are randomly distributed at position $x(t)$ in a multidimensional solution space (dimensionality D) and given an initial velocity $v(t)$ of random size and orientation.

(2) Each agent obtains the evaluation value of the target function at the current position $x(t)$.

(3) The evaluation value is compared with the best solution x_{pbest} that the agent has determined thus far. If the current value is better than x_{pbest}, it is updated to the current position $x(t)$.

(4) The evaluation values are compared among all agents, and the position of the best agent determined thus far is updated as x_{gbest}.

(5) The update of v and x for each agent follows Eq. (1):

$$\begin{cases} v_{id}(t+1) = w(t)v_{id} \\ \qquad\qquad + C_1\text{rand}()[x_{\text{pbest},id} - x_{id}(t)] \\ \qquad\qquad + C_2\text{rand}()[x_{\text{gbest},d}(t) - x_{id}(t)], \\ x_{id}(t+1) = x_{id}(t) + v_{id}(t+1), \end{cases} \tag{1}$$

where w is the momentum term, rand() is a uniform random number between 0 and 1, i is a agent number, d is a dimensionality, and C_1 and C_2 are constants that determine the amplitude of the random number. The momentum term, w, decreases linearly with the number of updates within a certain range.

(6) Repeat steps (2)–(5) for each iteration.

Figure 1 shows an example of a moving agent in standard PSO. The agents move according to three vectors: the momentum term, global best, and local best.

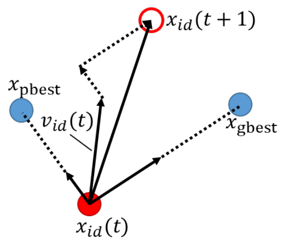

Figure 1: Updating position of agent in PSO.

2.2 DPSOP

In the proposed DPSOP, we change the velocity update equation in step (5) of standard PSO:

$$\begin{cases} v_{id}(t+1) = w(t)v_{id} \\ \qquad\qquad + C_1\dfrac{\cos[\phi_{id}(t)]+1}{2}[x_{\text{pbest},id} - x_{id}(t)] \\ \qquad\qquad + C_2\dfrac{\sin[\phi_{id}(t)]+1}{2}[x_{\text{gbest},d}(t) - x_{id}(t)], \\ x_{id}(t+1) = x_{id}(t) + v_{id}(t+1), \end{cases} \tag{2}$$

where ϕ is the phase of the cosine and sine wave with different initial values among agents, and

$$\phi_{id}(0) = \frac{2\pi}{ND} \times (i+d-2), \tag{3}$$

where N is a number of agents, and D is the number of dimensions. In addition, each value is $(i = 1,2,...N)$, $(d = 1,2,...D)$. The time step of the phase was $\frac{2\pi}{ND}$ with one iteration. Figure 2 shows an example of a moving agent in DPSOP. The agents move according to three vectors: the momentum term, global best, and local best. Agents move differently depending on their phases. In general, when the agents are strongly attracted to the global best, PSO has a high convergence, and when the agents are strongly attracted to the local best, PSO has a high escaping ability from the local minimum. In DPSOP, the agent is strongly attracted to the global best at $\phi = 0$, and strongly attracted to the local best at $\phi = \frac{\pi}{2}$.

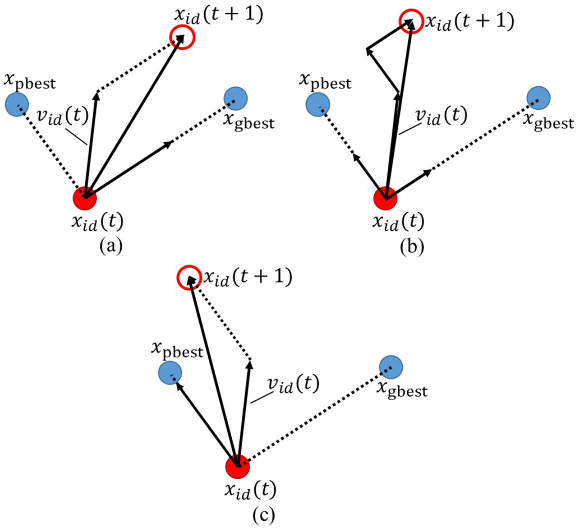

Figure 2: Updating position of agent in DPSOP. (a) Moving vector at $\phi = 0$. (b) Moving vector at $\phi = \frac{\pi}{4}$. (c) Moving vector at $\phi = \frac{\pi}{2}$.

3 Experiments

We compared the performance of DPSOP with standard PSO using five types of benchmark functions: the sphere function (f_1), Rosenbrock function (f_2), Rastrigin function (f_3), the Ackley function (f_4), and the Styblinski and Tang function (f_5) [17]. The minimum costs of f_1–f_4 were zero, and the minimum cost of f_5 was $-39.16599 \times D$. Equations(4)-(8) express these benchmark functions:

$$f(x_1 \cdots x_D) = \sum_{d=1}^{D} x_i^2. \tag{4}$$

$$f(x_1 \cdots x_{D-1}) = \sum_{d=1}^{D} (100(x_{d+1} - x_d^2)^2 + (x_d - 1)^2). \tag{5}$$

$$f_3(x_1 \cdots x_D) = 10n + \sum_{d=1}^{D} (x_d^2 - 10\cos(2\pi x_d)). \tag{6}$$

$$f_2(x_1 \cdots x_D) = 20 - 20\exp\left(-0.2\sqrt{\frac{1}{d}\sum_{i=1}^{D} x_d^2}\right) + e - \exp\left(\frac{1}{d}\sum_{i=1}^{d}\cos(2\pi x_d)\right). \tag{7}$$

$$f_5(x_1 \cdots x_D) = \frac{\sum_{d=1}^{D} x_d^4 - 16x_d^2 + 5x_d}{2}. \tag{8}$$

Table 1 lists the simulation conditions and parameters [18][19]. The average error, minimum error, maximum error, standard deviation, and average calculation time were obtained from 10 trials. The settings for the initial position and velocity of each PSO model were the same.

3.1 Results

The experiment was divided into two conditions; C_1 and C_2 are parameters that determine the moving distance of the particles.

Table 1: Simulation Conditions

Trial	10
Iteration	10000
Population	30 and 100
Dimensions	30 and 100
w	1.0-0.6
C_1	1.6, 2.0
C_2	1.6, 2.0

3.1.1 $C_1 = C_2 = 1.6$

Tables 2–6 present the simulation results for the five PSOs with $C_1 = C_2 = 1.6$. Overall, the proposed DPSOP (M_2) obtains better solutions than standard PSO (M_1). For a populations of $N = 100$ and dimensions $D = 30$, standard PSO determined better solutions than the proposed DPSOP for f_3–f_5.

Table 2: Simulation Results of Sphere Function ($C_1 = C_2 = 1.6$).

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	2.63E+01	7.97E+00	4.70E+01	1.18E+00	**4.43E+00**
		M_2	**5.48E-02**	**4.10E-03**	**2.29E-01**	**6.65E-02**	4.80E+00
		M_3	9.98E-01	2.24E-01	2.16E+00	5.65E-01	4.78E+00
		M_4	1.68E+00	4.63E-01	5.37E+00	1.35E-01	4.77E+00
		M_5	2.31E+00	4.23E-01	4.54E+00	1.20E+00	4.80E+00
	100	M_1	1.76E+00	2.72E-3	7.12E+00	2.29E+00	**1.43E+01**
		M_2	**6.21E-04**	**9.03E-05**	**2.06E-03**	**5.81E-04**	1.49E+01
		M_3	1.57E-02	3.86E-03	5.98E-02	1.55E-02	1.48E+01
		M_4	1.30E-02	9.14E-04	3.51E-02	1.07E-02	1.52E+01
		M_5	2.49E-02	1.97E-03	1.61E-01	4.57E-02	1.51E+01
100	30	M_1	4.03E+02	3.27E+02	4.65E+02	4.06E+01	1.42E+01
		M_2	**2.78E+01**	**1.96E+01**	**3.71E+01**	**5.78E+00**	1.42E+01
		M_3	4.27E+01	2.36E+01	5.65E+01	9.28E+00	1.42E+01
		M_4	6.91E+01	4.01E+01	1.05E+02	1.80E+01	1.42E+01
		M_5	7.39E+01	3.96E+01	1.02E+02	2.13E+01	1.43E+01
	100	M_1	1.75E+02	1.32E+02	2.12E+02	2.75E+01	4.68E+01
		M_2	**7.55E+00**	**5.65E+00**	**1.10E+01**	**1.79E+00**	**4.58E+01**
		M_3	1.26E+01	8.64E+00	1.62E+01	2.49E+00	4.56E+01
		M_4	1.72E+01	1.48E+01	2.09E+01	1.96E+00	4.66E+01
		M_5	1.51E+01	1.05E+01	1.99E+01	3.20E+00	4.65E+01

3.1.2 $C_1 = C_2 = 2.0$

Tables 7–11 present the simulation results for the five PSOs when $C_1 = C_2 = 2.0$. PSO with cos and cos implies that we used only the cosine for the length of the agent moving equation, Eq. (2). The results of the proposed DPSOP are overall better than those of the other methods. For the Ackley function with $D = 30$ and $N = 100$, the result of PSO was better than that of the DPSOP. The Ackley function has many local minima and narrow optimal-

Table 3: Simulation Results of Rosenbrock Function ($C_1 = C_2 = 1.6$).

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	2.63E+01	7.97+00	4.70E+00	4.70E+01	**4.43E+00**
		M_2	**5.48E-02**	**4.10E-03**	**2.29E-01**	**6.65E-02**	4.80E+00
		M_3	9.98E-01	2.24E-01	2.16E+00	5.56E-01	4.78E+00
		M_4	1.68E+00	4.63E-01	5.37E+00	1.35E+00	4.77E+00
		M_5	2.31E+00	4.23E-01	4.54E+00	1.20E+00	4.80E+00
	100	M_1	1.76E+00	2.72E-03	7.12E+00	2.29E+00	**1.43E+01**
		M_2	**6.21E-04**	**9.03E-05**	**2.06E-03**	**5.81E-04**	1.49E+01
		M_3	1.57E-02	3.86E-03	5.98E-02	1.55E-02	1.48E+01
		M_4	1.30E-02	3.86E-03	5.98E-02	1.55E-02	1.48E+01
		M_5	2.49E-02	1.97E-03	1.61E-01	4.57E-02	1.51E+01
100	30	M_1	4.03E+02	3.27E+02	4.65E+02	4.06E+01	**1.42E+01**
		M_2	**2.78E+01**	**1.96E+01**	**3.71E+01**	**5.78E+00**	1.42E+01
		M_3	4.27E+01	2.36E+01	5.65E+01	9.28E+00	1.42E+01
		M_4	6.91E+01	4.01E+01	1.05E+02	1.80E+01	1.42E+01
		M_5	7.39E+01	3.96E+01	1.02E+02	2.13E+02	1.43E+01
	100	M_1	1.75E+02	1.32E+02	2.12E+02	2.75E+01	**4.68E+01**
		M_2	**7.55E+00**	**5.65E+00**	**1.10E+01**	**1.79E+00**	4.58E+01
		M_3	1.26E+01	8.64E+00	1.62E+01	2.49E+00	4.56E+01
		M_4	1.72E+01	1.48E+01	2.09E+01	1.96E+00	4.66E+01
		M_5	1.51E+01	1.05E+01	1.99E+01	3.20E+00	**4.65E+01**

Table 4: Simulation Results of Rastrigine Function ($C_1 = C_2 = 1.6$).

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	1.12E+02	5.49E+01	1.76E+02	3.72E+02	**1.67E+01**
		M_2	7.44E+01	3.84E+01	1.29E+02	**2.38E+01**	1.68E+01
		M_3	**6.77E+01**	**3.81E+01**	**9.29E+01**	1.59E+01	1.68E+01
		M_4	9.26E+01	6.46E+01	1.28E+02	1.71E+01	1.67E+01
		M_5	7.83E+01	5.20E+01	1.12E+02	2.25E+01	1.69E+01
	100	M_1	**1.61E+01**	**9.95E-01**	**3.39E+01**	1.21E+01	**5.84E+01**
		M_2	4.64E+01	2.99E+01	6.87E+01	**1.11E+01**	5.93E+01
		M_3	6.11E+01	3.30E+01	8.70E+01	1.66E+01	5.91E+01
		M_4	6.82E+01	3.99E+01	9.26E+01	1.72E+01	5.96E+01
		M_5	5.79E+01	4.00E+01	8.07E+01	1.35E+00	5.89E+01
100	30	M_1	1.10E+03	9.81E+02	1.22E+03	6.75E+01	5.75E+01
		M_2	**4.37E+02**	3.77E+02	**5.46E+02**	**5.24E+01**	**5.69E+01**
		M_3	5.23E+02	**3.76E+02**	6.03E+02	6.16E+01	5.83E+01
		M_4	6.06E+02	5.12E+02	6.77E+02	4.83E+01	5.75E+01
		M_5	6.01E+02	4.96E+02	6.99E+02	5.68E+01	5.77E+01
	100	M_1	5.60E+02	4.27E+02	6.46E+02	5.90E+01	1.90E+02
		M_2	**3.22E+02**	**2.17E+02**	**3.76E+02**	4.90E+01	1.87E+02
		M_3	3.68E+02	3.04E+02	4.14E+02	**3.92E+01**	1.87E+02
		M_4	4.36E+02	3.76E+02	4.96E+02	3.95E+01	**1.84E+02**
		M_5	4.15E+02	2.92E+02	5.25E+02	7.06E+01	1.89E+02

Table 5: Simulation Results of Ackley Function ($C_1 = C_2 = 1.6$).

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	1.20E+01	**8.29E+00**	1.69E+01	2.69E+00	**1.71E+01**
		M_2	**1.14E+01**	1.01E+01	**1.28E+01**	**7.12E-01**	1.74E+01
		M_3	**1.14E+01**	9.13E+00	1.37E+01	7.12E+01	1.74E+01
		M_4	1.24E+01	9.13E+00	1.37E+01	1.50E+00	1.73E+01
		M_5	1.27E+01	8.77E+00	1.39E+01	1.34E+01	1.70E+01
	100	M_1	**5.56E-14**	**2.13E-14**	1.56E-13	3.67E+01	5.66E+01
		M_2	7.84E+00	5.98E+00	9.42E+00	1.01E+00	5.74E+01
		M_3	8.14E+00	6.84E+00	9.90E+01	**9.25E-01**	**5.51E+01**
		M_4	8.15E+00	6.60E+00	1.02E+01	1.04E+00	5.63E+01
		M_5	8.44E+00	6.66E+00	1.01E+01	1.17E+00	5.52E+01
100	30	M_1	1.86E+01	1.75E+01	1.92E+01	**4.95E-01**	5.26E+01
		M_2	1.45E+01	1.36E+01	1.58E+01	6.54E-01	**5.25E+01**
		M_3	**1.43E+01**	**1.30E+01**	**1.56E+01**	8.10E-01	5.30E+01
		M_4	1.57E+01	1.45E+01	1.71E+01	6.59E-01	5.32E+01
		M_5	1.58E+01	1.43E+01	1.67E+01	7.13E-01	5.42E+01
	100	M_1	1.53E+01	1.13E+01	1.71E+01	1.92E+00	1.64E+02
		M_2	**1.17E+01**	1.08E+01	**1.25E+01**	**5.33E-01**	1.67E+02
		M_3	**1.17E+01**	**1.01E+01**	1.30E+01	8.69E-01	**1.62E+02**
		M_4	1.31E+01	1.15E+01	1.44E+01	9.09E-01	1.63E+02
		M_5	1.31E+01	1.11E+01	1.49E+01	1.10E+00	**1.62E+02**

Table 6: Simulation Results of Styblinski-Tang Function ($C_1 = C_2 = 1.6$).

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	-9.64E+02	**-1.09E+03**	-8.79E+02	5.57E+01	1.17E+01
		M_2	**-9.97E+02**	-1.05E+03	**-9.48E+02**	2.97E+01	1.18E+01
		M_3	-9.88E+02	-1.04E+03	-9.23E+02	4.28E+01	**1.16E+01**
		M_4	-9.73E+02	-1.07E+03	-9.28E+02	4.28E+01	**1.16E+01**
		M_5	**-9.80E+02**	-1.01E+03	-9.45E+02	**2.18E+01**	1.19E+01
	100	M_1	**-1.16E+03**	**-1.18E+03**	**-1.12E+03**	**1.75E+01**	**3.84E+01**
		M_2	-1.00E+03	-1.04E+03	-9.49E+02	2.34E+01	**3.84E+01**
		M_3	-1.04E+03	-1.10E+03	-9.91E+02	3.03E+01	3.92E+01
		M_4	-1.02E+03	-1.06E+03	-9.60E+02	3.07E+01	3.88E+01
		M_5	-1.03E+03	-1.09E+03	-9.86E+02	3.34E+01	3.87E+01
100	30	M_1	-1.98E+03	-2.26E+03	-1.68E+03	1.60E+02	**3.75E+01**
		M_2	**-2.72E+03**	**-2.80E+03**	**-2.60E+03**	6.47E+01	3.79E+01
		M_3	-2.59E+03	-2.68E+03	-2.48E+03	**5.61E+01**	3.78E+01
		M_4	-2.58E+03	-2.75E+03	-2.49E+03	7.87E+01	3.79E+01
		M_5	-2.56E+03	-2.71E+03	-2.44E+03	7.19E+01	3.81E+01
	100	M_1	**-2.95E+03**	**-3.13E+03**	-2.58E+03	1.59E+02	**1.26E+02**
		M_2	-2.88E+03	-2.92E+03	**-2.80E+03**	**3.19E+01**	1.27E+02
		M_3	-2.80E+03	-2.93E+03	-2.66E+03	7.46E+01	1.27E+02
		M_4	-2.79E+03	-2.93E+03	-2.62E+03	9.88E+01	**1.26E+02**
		M_5	-2.81E+03	-2.89E+03	-2.69E+03	5.81E+01	**1.26E+02**

solution areas. Standard PSO, which involves a large number of agents, can search widely in the solution space because the moving length is randomly determined. Thus, standard PSO can determine a narrower optimal solution area than that determined by DPSOP. In other cases, the DPSOP has two characteristic phases: $\phi = 0$ and $\phi = \frac{\pi}{2}$. At $\phi = 0$, the agents of the DPSOP is affected only by the global best solution; thus, the agents are rapidly gathered to the global best. Therefore, the DPSOP can search the current global best solution in detail. At $\phi = \frac{\pi}{2}$, the agent of the DPSOP is affected only by the local best solution; thus, it moves away from the global best. We believe that the two characteristic phases are efficient in searching for better solutions and escaping from the local minimum. In fact, the DPSOP can determine better solutions in both unimodal and multimodal functions.

Table 7: Simulation Results of Sphere Function ($C_1 = C_2 = 2.0$).

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	2.94E+01	7.07E+00	4.08E+01	9.38E+00	**4.20E+00**
		M_2	**3.33E-02**	**4.31E-03**	**6.42E-02**	**2.23E-02**	4.53E+00
		M_3	8.31E-01	1.97E-01	2.55E+00	6.98E-01	4.51E+00
		M_4	2.02E+00	4.04E-01	3.73E+00	9.76E-01	4.49E+00
		M_5	1.94E+00	5.01E-01	6.22E+00	1.57E+00	4.48E+00
	100	M_1	3.16E+00	1.94E-129	9.86E+00	3.52E+00	**1.36E+01**
		M_2	**1.54E-03**	**1.81E-06**	**5.17E-03**	**1.68E-03**	1.41E+01
		M_3	2.95E-02	5.40E-03	6.96E-02	2.15E-02	1.41E+01
		M_4	1.31E-02	1.22E-03	2.65E-02	8.57E-03	1.43E+01
		M_5	2.27E-02	4.79E-03	6.87E-02	1.93E-02	1.41E+01
100	30	M_1	4.47E+02	3.90E+02	5.90E+02	5.90E+01	1.34E+01
		M_2	**2.29E+01**	**1.32E+01**	**3.89E+01**	**6.91E+00**	1.34E+01
		M_3	4.50E+01	3.39E+01	6.40E+01	8.48E+00	1.34E+01
		M_4	7.66E+01	5.80E+01	1.03E+02	1.49E+01	1.34E+01
		M_5	6.09E+01	4.93E+01	8.65E+01	1.08E+01	1.34E+01
	100	M_1	1.72E+02	1.37E+02	2.22E+02	2.37E+01	4.45E+01
		M_2	**7.08E+00**	**4.46E+00**	**1.01E+01**	**1.50E+00**	**4.41E+01**
		M_3	1.45E+01	1.05E+01	1.86E+01	3.03E+00	4.41E+01
		M_4	1.68E+01	9.53E+00	2.15E+01	3.26E+00	4.42E+01
		M_5	1.51E+01	8.50E+00	2.04E+01	3.64E+00	4.43E+01

3.1.3 Discussions

Table 12 compares the average error of the proposed DPSOP with that of standard PSO. "W" in Table 12indicates when the average error of the proposed DPSOP is lower than that of standard PSO. The performance of the proposed DPSOP is better than that of standard PSO for every benchmark function when the population is 30. The proposed method can efficiently search for solutions using a few particles. This is because the distance that was moved towards the global best changes periodically, which allows for a wider solution search. When the number of populations is 100, standard PSO improved the search ability. Many particles are distributed to the search space, and the particles rapidly gather to the global best position. The gathering speed of DPSOP to the global best is slower than that of standard PSO; thus, the improvement ratio of DPSOP is lower than that of standard PSO by population increasing.

Table 8: Simulation Results of Rosenbrock Function $(C_1 = C_2 = 2.0)$.

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	1.12E+03	6.82E+02	2.55E+03	5.17E+02	**1.27E+01**
		M_2	**2.70E+01**	**2.43E+01**	**3.10E+01**	**2.17E+00**	1.29E+01
		M_3	5.21E+01	3.05E+01	7.29E+01	1.34E+01	1.28E+01
		M_4	7.31E+01	3.65E+01	1.29E+02	2.88E+01	1.28E+01
		M_5	8.62E+01	3.89E+01	1.64E+02	3.69E+01	1.29E+01
	100	M_1	8.10E+01	1.21E-01	2.91E+02	8.88E+01	**4.07E+01**
		M_2	**2.33E+01**	**2.20E+01**	**2.71E+01**	**1.53E+00**	4.26E+01
		M_3	2.62E+01	2.46E+01	2.83E+01	1.27E+00	4.29E+01
		M_4	2.38E+01	2.18E+01	2.66E+01	1.39E+00	4.28E+01
		M_5	2.50E+01	2.36E+01	2.88E+01	1.53E+00	4.26E+01
100	30	M_1	1.91E+04	1.61E+04	2.25E+04	2.05E+03	**4.22E+01**
		M_2	**5.42E+02**	**3.97E+02**	**6.59E+02**	**7.49E+01**	4.24E+01
		M_3	8.31E+02	5.86E+02	1.09E+03	1.80E+02	4.28E+01
		M_4	1.72E+03	1.21E+03	2.39E+03	3.04E+02	4.25E+01
		M_5	1.95E+03	1.29E+03	3.27E+03	5.70E+02	4.23E+01
	100	M_1	4.65E+02	3.39E+02	6.93E+02	1.15E+02	**1.42E+02**
		M_2	**2.60E+02**	**1.93E+02**	**3.04E+02**	**3.24E+01**	1.46E+02
		M_3	3.19E+02	2.38E+02	3.86E+02	4.54E+01	1.46E+02
		M_4	4.69E+02	3.45E+02	6.11E+02	8.86E+01	1.43E+02
		M_5	4.27E+02	3.01E+02	6.05E+02	8.63E+01	**1.42E+02**

Table 9: Simulation Results of Rastrigine Function $(C_1 = C_2 = 2.0)$.

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	1.23E+02	9.46E+01	1.63E+02	2.23E+01	**1.56E+01**
		M_2	**6.38E+01**	**3.41E+01**	**8.73E+01**	**1.63E+01**	1.59E+01
		M_3	6.93E+01	3.64E+01	1.06E+02	2.16E+01	1.59E+01
		M_4	8.67E+01	4.59E+01	1.28E+02	2.77E+01	1.59E+01
		M_5	8.67E+01	6.37E+01	1.04E+02	1.37E+01	1.60E+01
	100	M_1	**1.22E+01**	**9.95E-01**	**2.52E+01**	**8.52E+00**	5.25E+01
		M_2	5.78E+01	3.68E+01	8.86E+01	1.75E+01	**5.22E+01**
		M_3	4.60E+01	1.61E+01	7.48E+01	1.62E+01	5.28E+01
		M_4	6.93E+01	5.38E+01	8.82E+01	9.88E+00	5.26E+01
		M_5	7.07E+01	4.13E+01	1.00E+02	1.62E+01	5.26E+01
100	30	M_1	1.13E+03	1.01E+03	1.20E+03	5.85E+01	5.14E+01
		M_2	**4.24E+02**	**3.53E+02**	**4.80E+02**	**4.79E+01**	5.14E+01
		M_3	5.36E+02	4.25E+02	6.91E+02	7.49E+01	5.16E+01
		M_4	6.03E+02	4.79E+02	7.36E+02	7.39E+01	5.26E+01
		M_5	6.33E+02	5.44E+02	7.27E+02	6.39E+01	**5.11E+01**
	100	M_1	5.59E+02	4.53E+02	6.58E+02	6.08E+01	**1.70E+02**
		M_2	**2.97E+02**	**2.46E+02**	**3.60E+02**	**3.83E+01**	**1.72E+02**
		M_3	3.70E+02	3.33E+02	4.32E+02	3.43E+01	1.70E+02
		M_4	4.44E+02	3.72E+02	5.08E+02	4.69E+01	1.71E+02
		M_5	4.19E+02	3.23E+02	4.49E+02	3.43E+01	1.71E+02

Table 10: Simulation Results of Ackley Function ($C_1 = C_2 = 2.0$).

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	1.09E+01	**6.13E+00**	1.56E+01	3.08E+00	**1.48E+01**
		M_2	**1.04E+01**	8.97E+00	**1.19E+01**	**8.84E-01**	1.55E+01
		M_3	1.03E+01	8.55E+00	1.29E+01	1.36E+00	1.54E+01
		M_4	1.26E+01	9.25E+00	1.59E+01	1.82E+00	1.55E+01
		M_5	1.21E+01	1.00E+01	1.43E+01	1.32E+00	1.53E+01
	100	M_1	**1.58E-13**	**2.13E-14**	**1.17E-12**	**3.39E-13**	**4.91E+01**
		M_2	8.28E+00	5.79E+00	1.07E+01	1.53E+00	4.92E+01
		M_3	8.31E+00	5.88E+00	1.03E+01	1.60E+00	5.06E+01
		M_4	8.77E+00	6.26E+00	1.04E+01	1.18E+00	4.97E+01
		M_5	8.79E+00	6.33E+00	1.24E+01	1.62E+00	5.16E+01
100	30	M_1	1.88E+01	1.78E+01	1.93E+01	4.18E-01	**4.78E+01**
		M_2	**1.41E+01**	**1.34E+01**	**1.47E+01**	**3.72E-01**	4.79E+01
		M_3	1.43E+01	1.36E+01	1.53E+01	5.84E-01	4.82E+01
		M_4	1.61E+01	1.53E+01	1.71E+01	5.29E-01	4.80E+01
		M_5	1.59E+01	1.49E+01	1.69E+01	6.70E-01	**4.78E+01**
	100	M_1	1.55E+01	1.33E+01	1.80E+01	1.39E+00	1.54E+02
		M_2	**1.15E+01**	**1.01E+01**	**1.30E+01**	**7.30E-01**	**1.53E+02**
		M_3	1.14E+01	1.09E+01	1.20E+01	3.62E-01	1.54E+02
		M_4	1.29E+01	1.11E+01	1.48E+01	1.20E+00	1.53E+02
		M_5	1.21E+01	1.03E+01	1.34E+01	8.49E-01	**1.53E+02**

Table 11: Simulation Results of Styblinski-Tang Function ($C_1 = C_2 = 2.0$).

Dim.	Pop.	Model	Average	Minimum	Maximum	Std. Dev.	Time
30	30	M_1	-9.86E+02	**-1.08E+03**	-9.00E+02	5.06E+01	**1.16E+01**
		M_2	**-1.01E+03**	-1.06E+03	-9.48E+02	4.73E+01	1.20E+01
		M_3	-9.88E+02	-1.07E+03	-9.37E+02	3.48E+01	1.20E+01
		M_4	-1.00E+03	-1.06E+03	-9.16E+02	3.80E+01	1.20E+01
		M_5	**-1.01E+03**	-1.07E+03	**-9.50E+02**	**3.33E+01**	1.20E+01
	100	M_1	**-1.14E+03**	**-1.17E+03**	**-1.10E+03**	**2.28E+01**	**3.90E+01**
		M_2	-1.02E+03	-1.06E+03	-9.63E+02	2.94E+01	3.94E+01
		M_3	-1.03E+03	-1.08E+03	-9.77E+02	3.12E+01	3.95E+01
		M_4	-1.00E+03	-1.08E+03	-9.61E+02	3.62E+01	3.97E+01
		M_5	-1.01E+03	-1.06E+03	-9.75E+02	2.87E+01	3.96E+01
100	30	M_1	-1.86E+03	-2.11E+03	-1.62E+03	1.51E+02	3.90E+01
		M_2	**-2.77E+03**	**-2.83E+03**	**-2.68E+03**	**5.31E+01**	**3.86E+01**
		M_3	-2.59E+03	-2.73E+03	-2.46E+03	9.16E+01	3.89E+01
		M_4	-2.57E+03	-2.73E+03	-2.48E+03	6.85E+01	3.89E+01
		M_5	-2.61E+03	-2.84E+03	-2.46E+03	9.91E+01	3.93E+01
	100	M_1	**-2.92E+03**	**-3.18E+03**	-2.50E+03	1.79E+02	1.29E+02
		M_2	-2.91E+03	-2.99E+03	**-2.80E+03**	**6.07E+01**	**1.27E+02**
		M_3	-2.81E+03	-2.93E+03	-2.66E+03	8.37E+01	1.27E+02
		M_4	-2.83E+03	-2.98E+03	-2.71E+03	7.60E+01	1.26E+02
		M_5	-2.76E+03	-2.87E+03	-2.62E+03	8.99E+01	**1.27E+02**

Table 12: Comparison of results between the standard PSO (M_1) and the proposed PSO (M_2).

Dim.	Pop.	C_1, C_2	f_1	f_2	f_3	f_4	f_5
30	30	1.6	W	W	W	W	W
		2.0	W	W	W	W	W
	100	1.6	W	W	L	L	L
		2.0	W	W	L	L	L
100	1.6	30	W	W	W	W	W
		2.0	W	W	W	W	W
	100	1.6	W	W	W	L	L
		2.0	W	W	W	L	L

4 Conclusions

In this study, we proposed DPSOP, which introduces sine and cosine waves into the moving equation of the agent. The movement of agents in PSO is attracted to the global and local best solutions. In general, the length of the moving agent is determined at random. We introduced cosine and sine waves to the length of the agent moving towards the global and local best solutions, respectively. Thus, the length of the moving agent was determined in a deterministic manner. The proposed DPSOP has two characteristic phases: $\phi = 0$ and $\phi = \frac{\pi}{2}$. In these two phases, the agents are attracted only to the global or local best solutions. We compared the searching ability of DPSOP with that of standard PSO using five benchmark functions. From the results, DPSOP was determined to find a better solution than standard PSO. We believe that these two phases are efficient in terms of the solution searching ability of PSO.

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