

Revisiting the Algorithm RBSC-SubGen: Exploring its Limits and Improving its Convergence

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Abstract

This study focuses on the algorithm RBSC-SubGen, which is originally offered for generation of vocabulary decks but can potentially be applied for a variety of problems. We first study the resilience of RBSC-SubGen against various hyper-parameters by testing it under various constraints. Our results indicate that RBSC-SubGen is sensitive to subset size S , followed by desired RBSC coefficient ρ^* , permissible disparity ϵ and, finally, parent set size L . We then offer a simple modification for reducing the computational load of RBSC-SubGen and avoiding saturation. In particular, we offer to carry out an intermediate-level verification of the desired conditions. Additional tests show that the rate of saturation and the number of iterations decrease, whereas the disparity in the RBSC coefficient increases within acceptable limits.

Keywords: Simple difference formula, subset selection problem, rank-biserial correlation.

1 Introduction and Motivation

Real-world processes involve a certain degree of randomness. In modeling and estimation, the observations obtained from such processes are often treated as random variables and represented in terms of stochastic models.

The problem of *Ranking-and-selection* (R&S) from the computational statistics research field, focuses on processes involving random factors [1]. In particular, the objective is to choose the best of two (or sometimes more) processes (or items), which bear a certain degree of randomness, according to a given metric or measure. For reliably solving the R&S problem, it is necessary to take as many measurements as possible. However, this also implies that numerous physical experiments need to be carried out, which in certain cases may not be possible (e.g. vehicular traffic safety analysis). In addition, in certain other cases (e.g. pharmacological studies), they may cost a long time and/or financial resources.

In that matter, a convenient way of exploring process performance is to make *simulations* of stochastic models on computer platforms. Simulations enable the extension of

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the data set and investigation of complex phenomena without the need for analytical solutions [2]. Nevertheless, although they are easier to perform than physical experiments, they are not zero-cost either. Namely, since the stochastic models of real-world processes are often highly complex, simulating them may require significant time and computational resources.

Therefore, R&S algorithms try to find good solutions efficiently, namely, without making too many simulations. Most R&S algorithms target finding the (single) optimal process out of two or more processes according to a certain performance criteria [1]. Certain other R&S algorithms search for a subset of processes rather than a single one, which are termed as *subset selection* problems [3].

This study focuses on the subset selection problem and deploys the RBSC-based deck generation algorithm offered by [4] (henceforth, referred to as RBSC-SubGen) to solve it using a variety of hypothetical data sets under diverse constraints. Our contributions involve a detailed performance assessment of RBSC-SubGen and the proposal of a modification to enhance rate of saturations and number of necessary iterations.

To that end, by randomly sampling values out of a given distribution, we obtain various hypothetical data sets. In addition, by relaxing and tightening the desired specifications of the selected subset, we obtain a variety of constraints and assess the feasibility of the problem, rate of saturation, computational load and accuracy of the obtained solution. In this way, RBSC-SubGen is shown to be applicable to the subset selection problem and to be sensitive to subset size S , followed by desired RBSC coefficient ρ^* , permissible disparity ε and, finally, parent set size L . Next, we propose a simple modification on the original algorithm defined by [4] and carry out additional tests and observe the change in the aforementioned performance indicators. Our results show that the proposed modification helps to reduce the rate of saturation as well as the number of iterations for successful completion. Although the disparity in the desired value of the RBSC coefficient increases, it still stays within the permissible range, which means that it is not a real drawback.

2 Background and Related Work

Historically, R&S problems are often formulated in agricultural and clinical scenarios (e.g. grain yields, drug treatments) [5]. However, recently their application has expanded to a large domain involving judicial system [6], traffic safety [7], digital photo selection [8] among many others.

As mentioned in Section 1, while most of the R&S algorithms aim choosing the (single) best process [1, 9], certain others search for a subset of processes [10, 11, 3]. Subset selection is essential in evaluating the performance of complex systems using stochastic simulation. Namely, when optimizing system performance with such simulations, the final decision is taken by considering various simulation results. However, since simulations of complex systems are computationally expensive and time-consuming, it is desirable to first eliminate non-competitive designs and then study the remaining ones in detail [12], which can be addressed by subset selection.

Subset selection problems may have various purposes. For instance, if the measurements are known to contain noise, the solution chosen as the single best performing process may not be the *true best solution*, thus choosing a subset and studying them deeper may yield better results [13]. Another potential purpose of subset selection can be posthoc analysis. For instance, suppose that a group of patients is found to react to a test drug in a

certain way based on a set of low-cost medical exams. If one wants to investigate better the pharmacological effects with a reasonable cost, he/she may carry out additional high-cost medical exams on a proper (i.e. diverse as well as representative) subset of patients and achieve reliable results, while keeping cost-efficiency. Moreover, a second-stage algorithm or a follow-up simulation of a subsequent process can be tested in conjunction with a subset of solutions (of a preceding process) to achieve more accurate and realistic results (i.e. more robust against measurement noise or variations due to stochasticity).

The chosen subsets are often required to attain the largest or smallest performance measures and be homogeneous [12]. Nevertheless, depending on the application, various other conditions can be imposed as in [4] and [14]. Namely, in [4] the chosen subsets of vocabulary are desired to be uniform within the subset and diverse between the varying subsets, whereas in [14] the chosen subsets are desired to be as diverse and inclusive as possible to satisfy the ethical requirements on fairness.

Various R&S algorithms and subset selections approaches have been proposed including the indifference-zone approach (IZ), maximization of the probability of correct selection (PCS), optimal computing budget allocation (OCBA) and expected value of information (EVI) [15, 16, 9]. Since R&S requires pairwise comparison of performance metrics [17], some studies focus on this aspect and try to optimize R&S further by decreasing the number of pairwise comparisons [18].

3 Fundamentals of Simple Difference Formula

In this section, based on a simple hypothetical example, we will explain the simple difference formula of Kerby [19] and the computation of the RBSC coefficient.

Assume that we want to quantify the validity of the hypothesis that *Males are taller than females*. One way to do this is to compute the Rank-biserial correlation (RBSC). Essentially, RBSC defines the correlation concerning a dichotomous variable and a ranking variable. In the case of this example, the statement that *Males are taller than females* is a dichotomous variable (rated as either True or False). On the other hand, the height of each person in the sample population can easily be transformed into a ranking variable.

To evaluate the validity of the above-mentioned hypothesis based on RBSC, initially, the height of each pair of persons from opposite genders is compared. Suppose that a male person from the sample population has a height of h_m , where h_m is a real number. Similarly, let the height of a female person from the sample population be h_f . Let N_S stand for the number of evidence *supporting* the hypothesis, (i.e. the male is taller than the female). Then, N_S can simply be computed as

$$N_S = 0.5 \cdot \sum_{\forall m, f} (\text{sign}(h_m - h_f) + 1), \quad (1)$$

where $\text{sign}(\cdot)$ denotes signum. Similarly, let N_C represent the number of evidence *contradicting* the hypothesis (i.e. the female is taller than the male),

$$N_C = 0.5 \cdot \sum_{\forall m, f} (\text{sign}(h_f - h_m) + 1). \quad (2)$$

After obtaining the number of evidence supporting and contradicting the hypothesis as shown in Equations 1 and 2, we employ *Simple Difference Formula* proposed by Kerby [19], which yields the RBSC coefficient denoted with ρ .

Specifically, according to Kerby [19], the non-parametric correlation equals the simple difference between the proportion of “favorable” and “unfavorable” evidence, where favorable stands for the pairs supporting the hypothesis and unfavorable for the ones disagreeing with the hypothesis. Thus, RBSC coefficient ρ can be computed in explicit terms as,

$$\rho = \frac{N_S - N_C}{N_S + N_C}. \quad (3)$$

It is clear from Equation 3 that ρ is bounded in the range $[-1, 1]$. If the data are all favorable, then ρ will be exactly 1. On the contrary, if the data are all unfavorable, then ρ will be -1, whereas a correlation of 0 indicates an equal amount of favorable and unfavorable evidence. In that respect, if $\rho > 0$, it can be said that the hypothesis holds, and the level of validity can be measured by taking into account how close ρ is to 1.

3.1 Outline of the original RBSC-SubGen algorithm

Generally speaking, the purpose of RBSC-SubGen is to build a desired number (2 or possibly more) subsets (out of a large parent set). The parent set X involves certain items, each of which is associated with a *score*. The score can be an objective value (e.g. the height of a person as in the aforementioned example) or a subjective value (e.g. evaluation of a product in an online retail store). In the case of [4], the parent set is a lexicon, the subsets are vocabulary decks and the scores are the number of occurrences of the words in daily language.

Since the purpose of our work is not to adapt RBSC-SubGen to a certain data set or to try it with a certain set of constraints, in what follows, we avoid specifying the parent set (i.e. its items or how they are scored) and work on randomly generated sets from the standard normal distribution.

The essential condition of RBSC-SubGen is to build the subsets so as to attain the desired ranking relation, which is quantified in terms of rank-biserial correlation. In that respect, the inputs of RBSC-SubGen are composed of the parent set, the size of the output subsets, the desired RBSC coefficient ρ^* and a tolerable disparity on ρ^* , ε . Namely, any pair of subsets with an RSCB coefficient within the range $[\rho^* - \varepsilon, \rho^* + \varepsilon]$ is considered to be an output of RBSC-SubGen¹.

Specifically, [4] proposes building two initial (i.e. potential) subsets in an arbitrary manner and then updating them in an iterative fashion (by inserting and removing elements) such that the RBSC coefficient of the subsets converge to the desired value at each update. The algorithm ceases the update as soon as an RSCB coefficient within the acceptable range is attained and returns the subsets.

The most crucial step of RBSC-SubGen is the iterative update (see Lines 4-8 of Alg. 1). In realizing the update, firstly the current state of the subsets is assessed by calculating ρ (see Line 3 of Alg. 1). Then, based on the disparity of ρ in relation to ρ^* (i.e. whether it is too high or too low), the sort of necessary update (i.e. insertion/removal) is determined.

Suppose that between the subsets A and B , the one with lower scores (on average) is A . Further suppose that ρ_{AB} is lower than the desired value ρ^* than a maximum permissible disparity ε , i.e. $\rho^* - \rho_{AB} > \varepsilon$. In this case, A has to be updated such that an item with a relatively low score needs to be inserted into it and an item with a relatively high score needs to be removed (i.e. returned from A to X' , where X' is the remaining elements of the

¹In that respect, the solution is not unique.

Algorithm 1: RBSC-SubGen

Input: $X, S, \rho^*, \varepsilon, N_{\max}$
Output: $A, B \subset X$

- 1 **sample**($A \subset X, B \subset X \mid |A| = S, |B| = S$) // Build potential subsets
- 2 $X', \rho_{AB}, n := X - A - B, \text{RBSC}(A, B), 0$ // Available items X'
- 3 **while** $|\rho^* - \rho_{AB}| \geq \varepsilon \wedge n < N_{\max}$ **do**
- 4 $U_A, U_B = \text{GetRequiredUpdate}(A, B, \rho^*, \varepsilon)$
- 5 $A, X' = \text{UpdateSubset}(A, U_A, X')$
- 6 $B, X' = \text{UpdateSubset}(B, U_B, X')$
- 7 $\rho_{AB} = \text{RBSC}(A, B)$
- 8 $n = n + 1$
- 9 **return** A, B

Algorithm 2: GetRequiredUpdate

Input: $A, B, \rho^*, \varepsilon$
Output: U_A, U_B

- 1 $U_A, U_B, \rho_{AB} = 0, 0, \text{RBSC}(A, B)$
- 2 **if** $\rho_{AB} < \rho^* - \varepsilon$ **then** $U_A, U_B = -1, 1$
- 3 **else if** $\rho_{AB} > \rho^* + \varepsilon$ **then** $U_A, U_B = 1, -1$
- 4 **return** U_A, U_B

parent set X). Moreover, B has to be updated such that an item with a relatively high score needs to be inserted into it and an item with a relatively low score needs to be removed (i.e. returned from B to X'). We code the aforementioned update on A and B with $U_A = -1$ and $U_B = 1$. If ρ_{AB} is higher than the desired value ρ^* beyond ε (i.e. $\rho_{AB} - \rho^* > \varepsilon$), the updates will be opposite. On the other hand, if ρ_{AB} is within the acceptable range, no action is necessary (coded as $U_A = U_B = 0$).

The procedure illustrated in Alg. 3 details the update operation. Specifically, each update amounts to the removal of a single item from the subset and the insertion of a single item from the parent set into the subset. The item to be inserted or removed is chosen in an arbitrary fashion such that it provides a change on ρ_{AB} in the desired direction.

In Alg. 3, the random sampling of an item a with a high score from the subset A is denoted with **sample**($a \in A \mid s_a > m_A$), where s_a denotes the score of the item a and m_A is the median score of the items in A . Obviously, an arbitrary element a may not satisfy $s_a > m_A$, and the sampling operation may need to be repeated several times. In addition, in order to avoid extremely long running times (or divergence), a maximum number of iterations N_{\max} needs to be defined. After carrying out the update operation illustrated in Alg. 3 for both subsets A and B (see Lines 6, 7 in Alg. 1), the new value of RBSC coefficient is computed. If it turns out to be within the satisfactory range, the updates are ceased and the subsets are returned.

4 Exploring Convergence Properties of RBSC-SubGen

In this section, we quantify convergence properties of RBSC-SubGen in relation to a set of hyper-parameters.

Algorithm 3: UpdateSubset

Input: A, U_A, X'
Output: A, X'

- 1 **if** $U_A = -1$ **then** // Decrease scores in A
 | /* Pick a high-score item $a \in A$ and a low-score item $x \in X'$ */
- 2 | **sample**($a \in A, x \in X' \mid s_a > m_A, s_x < m_A$)
- 3 **else if** $U_A = 1$ **then** // Increase scores in A
 | /* Pick a low-score item $a \in A$ and a high-score item $x \in X'$ */
- 4 | **sample**($a \in A, x \in X' \mid s_a < m_A, s_x > m_A$)
- 5 $A = A \cup \{x\} \setminus \{a\}$ /* Move a from A to X' and x from X' to A */
- 6 $X' = X' \cup \{a\} \setminus \{x\}$
- 7 **return** A, X'

4.1 Hyper-parameters

The hyper-parameters of RBSC-SubGen are presented in Table 1. In what follows, we discuss the impact of each hyper-parameter on performance.

As the desired value of the RBSC coefficient ρ^* increases, there will be a larger margin between the scores of the items in the subsets. In addition, since the parent set is assumed to come from a normal distribution, the bulk of the items have medium scores. Retrieving an item from one of the two extremes advances ρ much more quickly in the desired direction, but the chance of sampling such a value is lower due to the normal distribution assumption. Thus, higher values of ρ^* indicate more challenging problems, which are likely to terminate after a larger number of iterations. If ρ^* is significantly high (or low), the algorithm is likely to get saturated, i.e. reach the maximum number of iterations N_{\max} without a successful completion.

Table 1: Hyper-parameters of the algorithm.

Variable	Description	Min	Max	Step size	Default
ρ^*	Desired value of RBSC coefficient	0.3	0.7	0.04	0.5
ε	Maximum permissible disparity on ρ^*	0.05	0.15	0.01	0.1
L	Size of the parent set ($ X $)	100	900	50	500
S	Size of subsets ($ A , B $)	100	500	20	300

In most R&S problems, the parent set X is collected through physical or simulated experiments and thus has a limited size. This implies that it may not be possible to achieve the exact value of ρ^* . In that case, it is necessary to define a maximum permissible value ε for the disparity on ρ^* , $\Delta\rho = |\rho^* - \rho|$. Provided that $\Delta\rho \leq \varepsilon$, ρ is considered to be sufficiently close to ρ^* . Clearly, if ε is low, a more accurate solution is desired, which may require a larger number of iterations and may even result in a failure in achieving convergence.

The challenge due to the limited size of the parent set deserves a devoted hyper-parameter, which we denote with L . If L is large, the algorithm is expected to have sufficient freedom in choosing the scores and is likely to converge (provided that the other hyper-parameters are not too strict). Nevertheless, if L is too small, even if the other hyper-parameters are not

strict, the algorithm is likely to terminate without achieving convergence.

The size of the output subsets S are also hyper-parameters of RBSC-SubGen. Note that although an increase in S has a limiting effect on the freedom of choosing the scores, provided that L is large enough, this effect can be expected to diminish. It is also interesting that an increase in S can somewhat make the effect of individual scores less significant. Namely, since the number of possible pairs is proportional to S^2 , as S increases, the number of possible pairs increases even with a larger margin². In that respect, the potential impact of a single insertion/removal decreases with increasing S .

4.2 Specifics of testing

To understand the limits of RBSC-SubGen, we tested it with several combinations of hyper-parameter values. Specifically, in our tests, each hyper-parameter takes the values at the given step sizes between the Min and Max values presented in Table 1.

In addition, in order to visualize the interplay of two hyper-parameters (i.e. the pair-wise relations), the values of these hyper-parameters are varied at intermediate steps as defined before, while the remaining two hyper-parameters are set to pre-determined (i.e. default) values. For instance, while examining the interaction between L and S , ρ^* and ε are fixed at 0.5 and 0.1, respectively (see Table 1). In specific, each default value is set to the median of the corresponding range.

If $L > 2 \cdot S$, there is always some item that is not yet assigned to any of the subsets. Thus, such items are available to update the subsets. However, in certain cases (e.g. too little freedom in sampling or too little impact of modifying a single item), the algorithm may go into an infinite loop. Therefore, it is necessary to limit the maximum number of iterations N_{\max} for identifying such bottlenecks (see Line-4 of Alg. 1).

In addition, in order to get stable results, the algorithm is executed I times for each combination of hyper-parameters. At each execution, a fresh parent set is generated randomly. In addition, concerning each execution, we compute the performance indicators described in Section 4.3 and report their average values in Sections 4.4~4.6.

4.3 Quantification of performance

The performance of RBSC-SubGen is assessed in terms of three indicators as (i) rate of saturation, (ii) number of iterations until convergence and (iii) disparity on desired RBSC coefficient (see Table 2).

Table 2: Performance indicators.

β_0	Rate of saturation
M_0	Number of iterations until successful completion
$\Delta\rho$	Actual disparity on ρ^*

If the algorithm reaches N_{\max} iterations without achieving $\Delta\rho < \varepsilon$, it is said to get *saturated*. In that respect, one of the indicators in assessing the performance of the algorithm is the *rate of saturation* β_0 , which is defined as the ratio of the number of saturated attempts

² In other words, the modification of an individual score may affect at most S cases.

to the number of all attempts (i.e. I executions). In that respect, β_0 quantifies how often RBSC-SubGen fails to build the subsets.

If two subsets are built in less than N_{\max} iterations successfully, the algorithm is said to *converge*. In addition, the number of iterations until convergence denoted with M_0 is considered to be another performance indicator. Specifically, M_0 expresses how quickly RBSC-SubGen builds the subsets³.

As mentioned in Section 3.1, ε defines the maximum permissible disparity on ρ^* . If two subsets are built successfully, ρ is certainly closer to ρ^* than ε . This actual value of disparity $\Delta\rho$ is considered as another performance indicator. The observed disparity $\Delta\rho$ ideally depends strictly on ε . In that respect, the reason for computing $\Delta\rho$ is for making sure that the maximum number of iterations N_{\max} is sufficiently high. In other words, if it can be observed that $\Delta\rho$ depends solely on ε , it can be claimed that the values β_0 and M_0 are likely to grasp the general characteristics, and additional test runs are not necessary.

4.4 Results concerning rate of saturation

If the algorithm ceases without yielding an outcome (i.e. it is saturated), there is no point in studying most performance indicators. In that respect, the rate of saturation β_0 is considered to be the most important marker of performance.

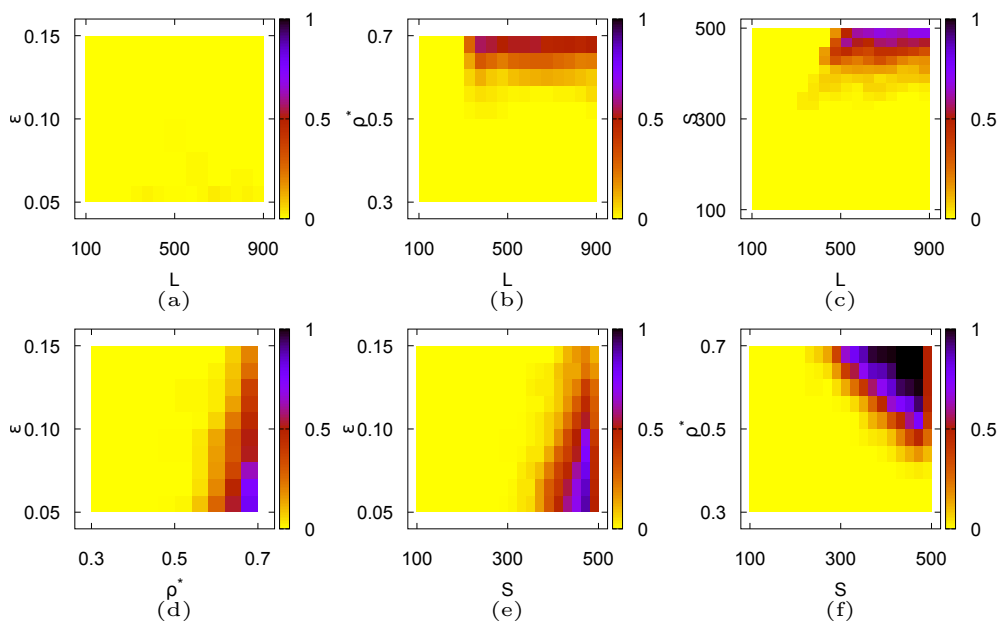


Figure 1: The effect of each possible hyper-parameter pair on β_0 .

Figure 1 illustrates the rate of saturation β_0 for each possible hyper-parameter pair. In particular, the hyper-parameter pair (ρ^*, S) often poses a challenge. Namely, there is a

³If the algorithm gets saturated, it does not converge and thus the number of iterations is not considered in computing M_0). Moreover, in the case that the algorithm gets constantly saturated for a certain set of hyper-parameters, M_0 is not available.

higher risk that the algorithm gets saturated when both the desired value of RBSC coefficient ρ^* and the size of target subsets S are large.

One may see in Figures 1-(d), (e) that large values of ρ^* pose a risk of saturation, also when they are coupled with small values of ε . In particular, the algorithm is observed to be more sensitive to ε than L (see also Figures 1-(b) and (d)).

In addition, a similar observation is valid also for S . Namely, when large values of S are coupled with small values of ε , a risk of saturation emerges. In addition, such a risk is more serious for ε than L (see Figures 1-(c) and (e)). Moreover, concerning the hyper-parameter pair of (ε, L) , the risk of saturation is not serious (see Figure 1-(a)).

4.5 Results concerning number of iterations

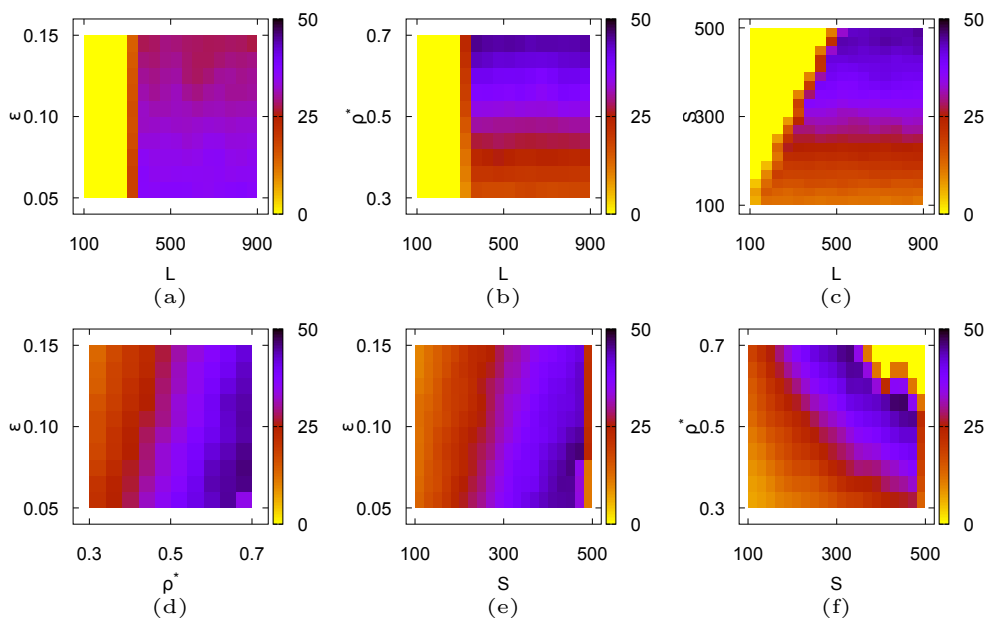


Figure 2: The effect of each possible hyper-parameter pair on M_0 .

Figure 2 illustrates our results concerning the number of iterations M_0 that it takes RBSC-SubGen to terminate successfully. In this figure, the yellow regions indicate either (i) the cases where the problem is not feasible (i.e. $L < 2 \cdot S$) or (ii) that all I runs are found to be saturated. If (i) is the case, there exists no solution due to an ill definition of the problem statement. However, if (ii) is the case, then one may try to improve the algorithm to overcome saturation. In Figures 2-(a), (b), (c), the yellow regions arise due to infeasibility. However, in Figures 2-(e), (f), they arise due to constant saturation.

Omitting the aforementioned cases (i) and (ii), we focus on successful executions. It is interesting that although the hyper-parameter pair of (ρ^*, S) has the largest number of saturations (see Figure 1-(f)), a higher number of iterations is required to solve most problems relating to the hyper-parameter pair of (ε, S) . This observation is somewhat valid also for the hyper-parameter pair of (ε, ρ^*) . As a consequence, the number of iterations is higher

for most (ε, S) and (ε, ρ^*) pairs, although saturation is not as common as in the case of (ρ^*, S) .

4.6 Results concerning disparity on desired RBSC coefficient

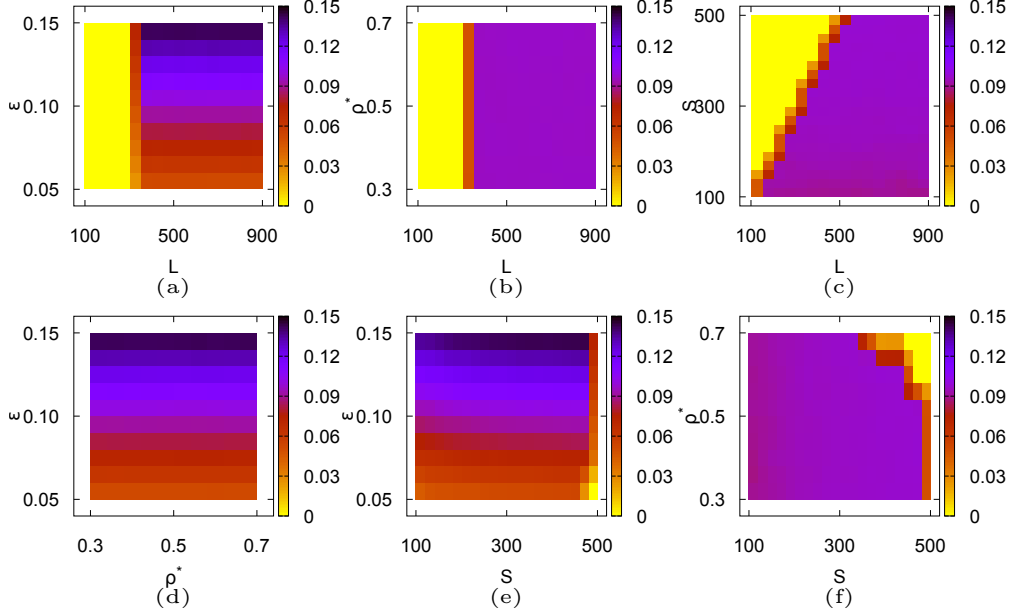


Figure 3: The effect of each possible hyper-parameter pair on $\Delta\rho$.

Finally, the evolution of disparity $\Delta\rho$ on the desired RBSC coefficient ρ^* is investigated relating to the six hyper-parameter pairs. It is observed in Figures 3-(a), (d), (e) that as long as ε is one of the hyper-parameters under investigation, $\Delta\rho$ depends solely on it and the other hyper-parameter under investigation does not introduce a prominent effect on $\Delta\rho$. This is not surprising since the termination of iterations is decided based on ε .

5 Improving Convergence Characteristics of RBSC-SubGen

In Alg. 1, the subsets are updated *in turn* at each iteration (see Lines 6, 7 in Alg. 1). Specifically, an insertion/removal is performed first on the subset with the lower ranks A , and then on the subset with higher ranks B . In that respect, the RBSC coefficient is modified already once after updating A . Therefore, it is worth checking whether it is within the acceptable range after this first update.

In that respect, we propose repeating the computation of ρ (see Line 8 of Alg. 1) between the updates of A and B (i.e. between Lines 6 and 7 of Alg. 1). This modification is expected to reduce β_0 and M_0 , whereas $\Delta\rho$ is expected not to be affected. We test the modified algorithm with the same rules as defined in Sections 4.1 and 4.2 and evaluate its performance with the same indicators defined in Section 4.3. We present the results concerning this modification in Sections 5.1~5.3.

5.1 Results concerning rate of saturation

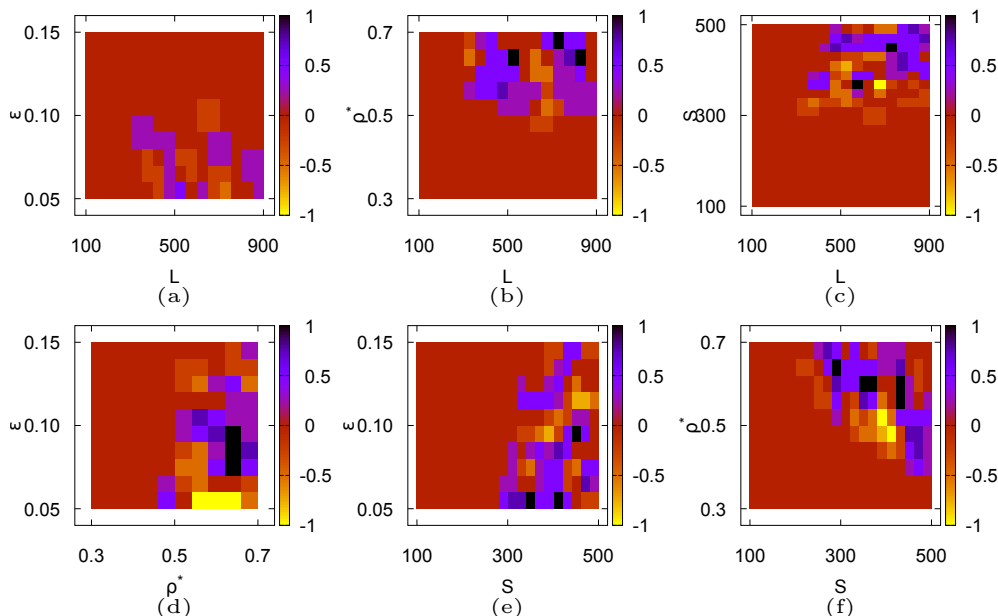


Figure 4: The change induced by the modified algorithm on β_0 for each possible hyper-parameter pair.

Figure 4 shows the change induced by the modified algorithm on β_0 for each possible hyper-parameter pair. Specifically, we run the modified algorithm I times and compare the number of saturated cases to the average number of saturated cases of the original algorithm. The improvement or deterioration is decided in a binary manner. Namely, if at a particular execution, the modified algorithm has less saturations than the original algorithm, the result is registered as 1, and otherwise, as a -1. This process is repeated I times and the averages corresponding to each hyper-parameter pair are illustrated in Figure 4. Excluding the cases where there is no difference, we can see in Figure 4 that positive values are in majority. In particular, we have improvement rates between 0.51 and 0.69.

5.2 Results concerning number of iterations

Figure 5 depicts the change induced by the modified algorithm on M_0 for each possible hyper-parameter pair. We carry out additional tests as explained in Section 5.1. We assess the change in performance in a binary manner. Namely, comparing the number of iterations of the modified algorithm concerning a particular hyper-parameter pair at a particular execution, to the average number of iterations of the original algorithm for the same pair, we register the result as a 1, if the former is larger, and otherwise, as a 0. We then take the average of the registered value for the I executions. Excluding the cases where there is no difference, we can see in Figure 5 that most M_0 values are positive, and only in a small number of cases the number of iterations increases. In particular, we have improvement rates between 0.57 and 0.69.

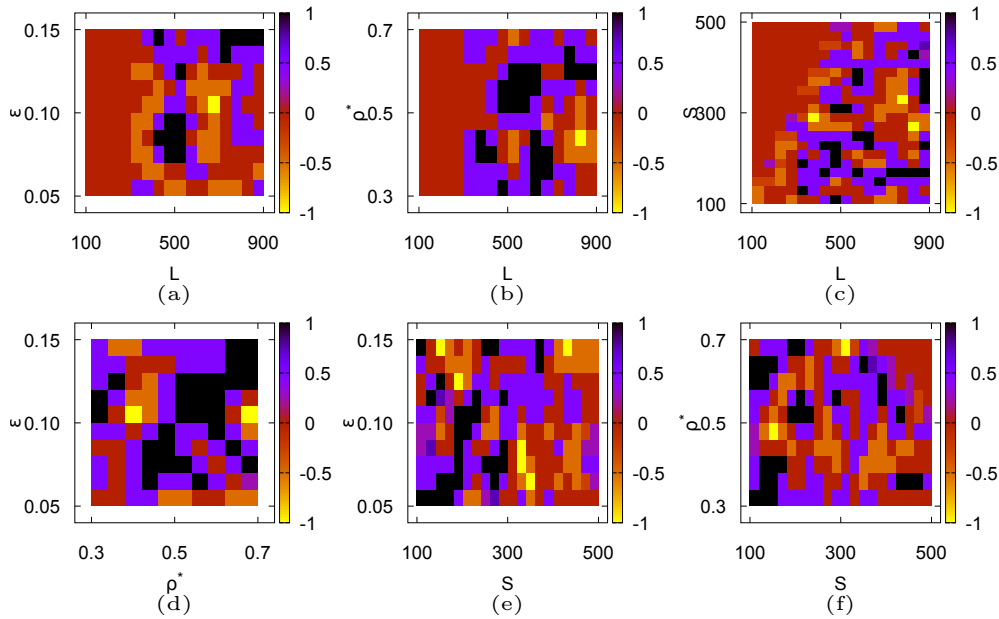


Figure 5: The change induced by the modified algorithm on M_0 for each possible hyperparameter pair.

5.3 Results concerning disparity on desired RBSC coefficient

Figure 6 depicts the change induced by the modified algorithm on $\Delta \rho$. The details of experimentation and the method of assessment of improvement or deterioration are similar to Sections 5.1 and 5.2. Excluding the cases where the problem is not feasible and where there is no difference, we can see that there is a fair degradation. Note that since ε is the determining factor on $\Delta \rho$ as shown in Section 4.6, it is not surprising that a smaller number of iterations lead to subsets with slightly larger disparity on ρ^* . It is important to note that the disparity is still smaller than the maximum permissible value ε .

6 Conclusions

This study focuses on RBSC-SubGen, which is originally designed for building vocabulary decks (out of a large corpus) with the desired level of word frequency relation. Exploiting the fact that this objective shares many common aspects with the generic subset selection problem, we tried RBSC-SubGen in generating subsets out of different hypothetical parent sets. We also imposed varying constraints on subset size, desired ranking relation, and permissible disparity. Our results indicate that RBSC-SubGen can be used in subset selection, provided that it is formulated as a ranking relation. In addition, RBSC-SubGen is found to be sensitive to subset size S , followed by desired RBSC coefficient ρ^* , permissible disparity ε and, finally, parent set size L . We then proposed a simple modification to the original algorithm such that the value of the RBSC coefficient is checked once after each update. Our results indicate to a decrease in the rate of saturation as well as the number of iterations,

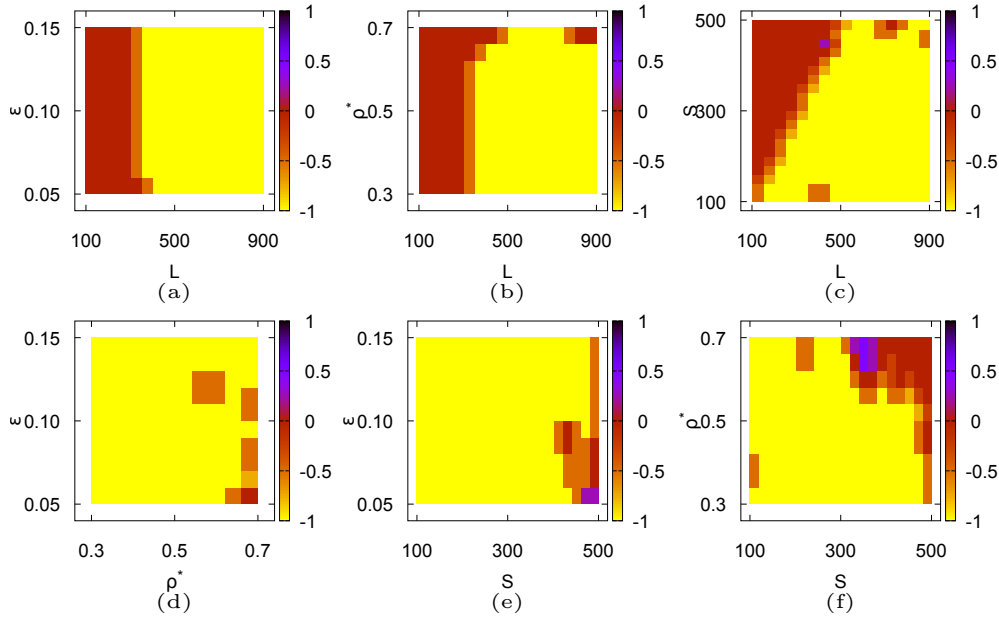


Figure 6: The change induced by the modified algorithm on $\Delta\rho$ for each possible hyperparameter pair.

whereas the disparity in the RBSC coefficient degrades slightly.

Acknowledgements

The codes developed for this study can be downloaded freely from [20].

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