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# Exploring the Critical Relationship between Precollege Preparation of Mathematics and College Graduation

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## Abstract

Completing a bachelor's degree is not only the primary goal for students, but also a crucial indicator of student achievement at each university. Despite the fact that factors influencing graduation rate have been investigated over the years, the scope, depth, and angles need to be further expanded. With a new angle and rarely studied pursue in this field, this study explored the factors that impacted students' graduation at a very high research university by employing a logistic regression model. The outcome of the study identified six factors that impacted student's graduation. Among the six factors, two most significant factors outstood, the difficulty level of the first mathematics course taken at the university, and whether students had to retake any of the six introductory science and mathematics courses requested by university. These two factors were related to their precollege preparation during their secondary school years. This study suggests that although more and more educational researchers and scholars had started to realize and raised these issues in the past, and governments and secondary schools had initiated programs to enhance students' mathematics proficiency, mathematics proficiency had not fundamentally improved and might remain a long-term challenge.

*Keywords:* mathematics precollege preparation, undergraduate graduation

## 1 Introduction

### 1.1 Objective of the Study

Completing a bachelor's degree is not only the primary goal for students, but also universally a crucial indicator of student achievement at each higher education institution. Since aggregating student academic achievement is a key factor for an institution's accreditation, effectiveness, assessment, and program evaluation, graduation is not only important for students themselves but also reflective of an institution's overall success.

Millea, Wills, Elder, and Molina [1] indicated a strong correlation between precollege

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preparation and students' academic performance in college with graduation being the final goal. Adelman [2] found "of all pre-college curricula, the highest level of mathematics one studies in secondary school has the strongest continuing influence on bachelor's degree completion." While he was a Senior Research Analyst at the U.S. Department of Education, Adelman initiated a longitudinal study in 1993 which tracked domestic secondary school students who were in the 10th grade in 1980 until they completed their bachelor's degree [2]. Inspired by Millea et al and Adelman's research on this topic, the approach of this study was to probe more specifically for the correlation between the courses taken by the students in their freshmen and sophomore years of college and their precollege preparation. The objectives of this study were to answer the following three questions:

1. Which factors impacted the students' graduation success?

2. What was the correlation between precollege preparation, especially in the subject of mathematics, and college graduation?

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3. Were the conclusions of Adelman's research [2] and similar conclusions of his later research in 2006 [3] validated at a very high research activity university as defined by Carnegie Classification of Institutions of Higher Education?

## 1.2 How is Graduation Rate Calculated?

Based on the latest information from the National Center for Education Statistics in the United States, the American average six-year graduation rate at four-year institutions for the 2013 freshman cohort was 63%. The corresponding rate was 62% at state institutions, 68% at private nonprofit institutions, and 26% at private for-profit institutions [4]. As defined by the U.S. Department of Education, the official method of calculating the graduation rate of four-year colleges is to track the students in a specific freshman cohort who have graduated within six years from the same institution. The students in the freshman cohort must be full-time, first-time, and degree-seeking. The term, "first-time students" indicates that they have never enrolled in any college courses after graduating from high school, no matter their age when they become students in a specific cohort. The freshman cohort used in this study was the group of students who began their college study in fall 2012 at the research university. Any student who graduated by summer 2018 from the same university was categorized as graduating on time.

The statistical method of calculating graduation rate was different for the United States compared to other countries of Organization for Economic Co-operation and Development (OECD). The graduation rate of the United States was based on students who began their college study in the fall semester of a specific year at a specific institution, while the graduation rate at other OECD countries was calculated based on specific age groups at each OECD country [5]. Figure 1 presents the graduation rates of other OECD countries. Due to the scope of data sources and the overall college environment discussed in this study, with caution the conclusions drawn were likely to be applicable to four-year colleges in the United States. However, with appropriate caution, the overall conclusion may be applicable to other OECD countries since it can be argued that the United States share similar background, such as culture, political system, and economic development level, with most of them; while the absolute rates vary, the two metrics should be highly correlated based on student age distribution. Since a major portion of new freshmen are 18-20 years old, graduating in six years is highly correctly with graduating by age 30. The latter would pick up a few more with gap years.

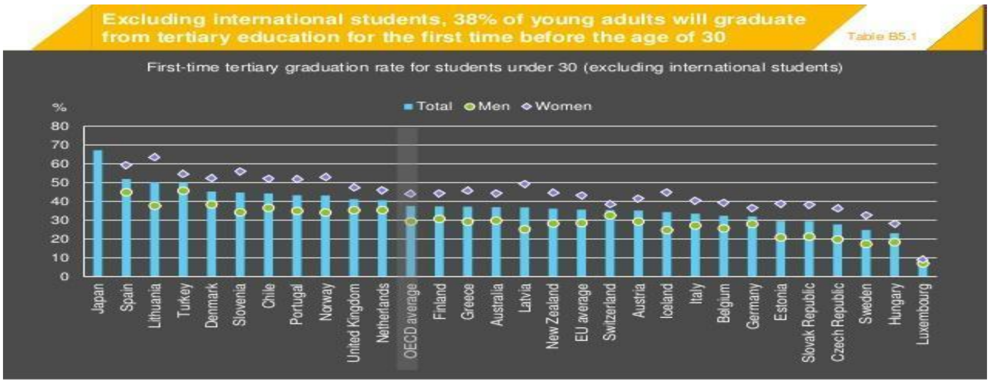


Figure 1: First-time tertiary graduation rate for students under 30
in other OECD countries [6]

## 2 Literature Review

### 2.1 Factors Impacting Graduation

Since many factors contribute to student graduation, college administration and different academic departments all sought to uncover why some students either were not able to graduate within six years or why they dropped out of college entirely. Among these reasons, Millea et al [1] believed that precollege preparation was one of the most crucial factors. They evaluated factors such as residential living, attendance programs, demographic attributes, average class size, and academic preparation by analyzing longitudinal, student-level data at one midsized state university in the southeastern United States from 1998 to 2004. The results of their study revealed that students who were better academically prepared from their secondary school studies had a higher chance of college graduation [1].

Chow [7] at the Rose-Hulman Institute of Technology emphasized the impact of mathematics precollege preparation on first to second year student retention. According to his study, about half of the students who left the institution after their first year of college study lacked necessary mathematics precollege preparation, demonstrated by either lower scores on the quantitative section of the SATs or the lack of completing any higher-level mathematics course beyond Algebra II during their high school period (High school is more popular name for secondary school as it has same meant of secondary school in this paper). Obviously, students who dropped out missed the opportunity of obtaining their degrees from their original institutions. Rasmussen and Ellis at San Diego State University analyzed the reasons why students of STEM (science, technology, engineering, and math) majors did not take the second calculus course after taking the first calculus course [8]. They found that 31.4% of the students believed their experience in Calculus I made them decide not to take Calculus II; 18.8% of the students said that they did not believe they understood the ideas of Calculus I well enough to take Calculus II; 11.5% of the students thought that their grade in Calculus I was not good enough for them to continue to Calculus II. Usually, students of STEM majors needed to take three sequential calculates. Since they did not take the Calculus II, it meant that they were no longer interested in or qualified studying the

STEM majors [8]. The results of the two studies above explained the relationship between students' academic performance in STEM majors and precollege preparation in mathematics.

Not only did students, who studied at regional state universities or private engineering college, face challenges mentioned in previous paragraphs when they took mathematics courses, but also the students at the research university had same obstacle. In their previous study, Author1, Author2, and Author3 [9] compared the students' academic performance during the first two collegiate years and precollege preparation. Among the predicators, SAT/ACT scores, race/ethnicity, gender, and Pell Grant status were not significant factors in impacting students' graduation in the fitted model, though but a variable related to precollege preparation in mathematics was significant. However, if the variable related to precollege preparation in mathematics was removed from the model, Pell Grant status became a significant factor in the fitted model that negatively affected graduation [10]. This can be interpreted to highlight that inadequate preparation in precollege mathematics was an even stronger factor impacting students' graduation negatively than Pell Grant status in the fitted model. While the challenge of a student's financial difficulty could be addressed by Pell Grants or other similar avenues of financial aid, inadequate preparation in precollege mathematics has no such quick fix. Therefore, it is a more serious challenge for students. Author1, Author2, and Author3' conclusions [9] coincided exactly with the research findings of Chow [7] and Rasmussen et al [8].

## 2.2 High School GPA

What are the components that make up college preparation? There are two well-recognized indicators of precollege preparation, high school GPA and standardized test scores [11]. While high school GPA was widely studied and spotlighted as an indicator of future college performance, this study consciously decided to exclude this metric from the model because of its unreliability as an indicator for several reasons.

First, compared with standardized test scores, the metric of high school GPA cannot universally be compared. Not only does each high school have its own curricula which can widely vary in terms of difficulty level, but each may have its own policy and methodology to calculate GPA. For example, some students' grades were weighted when taking Advanced Placement (AP) classes as 4.5 points for full score while some were weighted as 5 points. Some schools used the A to F grading scale while others used the 0-100 scale. Furthermore, for high schools without a weighted GPA, it would be problematic to fairly compare students with higher GPAs who take easier courses with students who take more difficult courses and end up with lower GPAs. In such cases, if offices of undergraduate admission only utilized high school GPA, they would overestimate or underestimate applicants' future college performance [12]. Furthermore, familiarity with such differences in grading may differ depending on the institution type. Undergraduate admission staff who work for prestigious private colleges may be more familiar with the grade systems of their traditional feeder high schools, in contrast to state universities who may face more serious challenges in identifying the various grading systems corresponding to each of their applicants since their applicants come from public and private high schools both in and out of state.

Another weakness of using high school GPA is that the overall curriculum of most high schools has more rigorous requirements in English but leaves more flexibility in courses related to mathematics and the science [2, 3, 13]. As a result, most students in four-year colleges may not face the same academic challenges when they take college-level English courses as they do in mathematics or science courses. In high school, only a few students take more advanced mathematics courses like Calculus or Linear Algebra in their junior or senior year, while most students only take up to Algebra II as their highest-level mathematics course [2, 3, 7, 13]. The content of Algebra II includes logarithm, exponent, and function instead of arrangement, combination, trigonometry, and analytic geometry. Even with simpler mathematics courses, students may not have successfully met the courses' learning objectives or even retained major knowledge by the time they attended their first college course. Unlike English, some students can begin college without a complete understanding of the contents that they had previously learned from their high school mathematics or science courses. Therefore, a higher high school GPA does not indicate that these students are college-ready in general, even if they choose majors that are not STEM related. As Cohen [14] said "High school grades are notoriously inflated and assessing the quality of a high school's curricula and grading policy is a time-consuming job." Adelman [2] analyzed high school students' academic performance and their college graduation rate. He identified three factors of Academic Resources that included high school curricula, standardized test scores, and high school GPA. Among them, high school curricula had a 41% weight, standardized test scores a 30% weight, and class rank/GPA a 29% weight. Thus, based on Adelman's analysis, high school GPA was the weakest factor in the category of Academic Resources.

## 2.3 Standardized Test Scores

Standardized test scores were another widely accepted indicator of college preparedness. They included the Scholastic Aptitude Test (SAT) and the American College Testing (ACT) in the United States. The SAT consisted of the English and mathematics sections, while the ACT had four components: English, Mathematics, Reading, and Science. Both were quite similar except the ACT's Science section, which measured the interpretation, analysis, evaluation, reasoning, and problem-solving skills required in the natural sciences [15]. Since SAT and ACT mathematics sections focused on logical thinking and mathematics skills at the middle school or even lower grade level, SAT Subject Tests, Math I and Math II, which focused on the contents of high school mathematics courses, were created to fill this knowledge gap. SAT Subject Tests in the Sciences included Biology, Chemistry, and Physics, separately. Only the most prestigious private colleges nationwide had required their applicants to submit two SAT Subject Tests up until early 2021 when the College Board abandoned all SAT Subject Tests [16]. Meanwhile, most state universities, except the nine universities of the University of California System [17], never had these requirements in the first place. Until 2012, 68 universities/colleges had required SAT Subject Tests, but these numbers kept shrinking until only 30 had such requirements in 2020. Since the market for SAT Subject Tests had all but disappeared, the College Board decided to eliminate them entirely from their offerings [16]. As a result, mathematics skills learned in high school cannot be readily measured using a national standard. Although this has not had any immediate impact on most colleges, sooner or later, this could lead to serious consequences for the most prestigious private colleges since the colleges neither employed the Mathematics Placement Test nor SAT Subject Math to measure their students' mathematics proficiency. However, what was more universally impactful was that even though standardized tests like the SAT and the ACT had been mandatory for applicants until a few years ago, with varying rationales, an increasing number of colleges have since become test optional [18]. Since the current study targeted the 2012 freshman cohort, almost all applicants submitted either their SAT or ACT scores when they applied to the research university. Therefore, standardized test scores were one independent variable used in the study. However, as discussed early, both SAT and ACT mathematics sections were inadequate on their own to evaluate students' mathematics proficiency.

## 2.4 High School Curricula

Adelman's research, titled *Answers in the Toolbox: Academic Intensity, Attendance Patterns, and Bachelor's Degree Attainment*, emphasized the role of students' high school curricula, specifically its intensity and quality. The study indicated that "the correlation of curricula with bachelor's degree attainment is also higher (0.54) than test scores (0.48) or class rank/GPA (0.44)" [2]. The updated version of that report was published by the United State Department of Education with the title *The Toolbox Revisited: Paths to Degree Completion from High School through College* in 2006. Among all the courses taken during high school, Adelman specifically spotlighted the role of advanced mathematics courses. He indicated that when students in high school had completed mathematics courses beyond Algebra II, the probability of the students eventually being awarded their bachelor's degrees was double [2]. Examples of more advanced mathematics courses included precalculus and calculus, among other similar options.

Besides rigorous high school curricula mentioned above by Adelman [2, 3], Rosenbaum [19], who was a professor of sociology, education, and social policy at Northwestern University, highly emphasized the role of homework during high school, since it is a core method of reaffirming what students learned in the classroom. The connection and logic between the rigorous curricula and homework are obvious. Without homework, how could students exercise, practice, and review what they learned in the classroom? Based on his study, without doing homework, students ended up with what is equivalent to 1.2 years less of education. On the other hand, after doing 15 or more hours of homework per week, students attained 1.5 more years of education. The theoretical 2.7 years spread in educational attainment would make a huge difference for any student [19].

Both Adelman [2, 3] and Rosenbaum [19] highlighted the importance of mathematics and homework in high school, and Adelman's research was even published by the U.S. Department of Education while being endorsed by the Secretary of Education, Margaret Spellings in 2006 [3]. However, high school students still did not pay adequate attention to their mathematics curricula and homework. Rosenbaum [19] summarized six misconceptions which contributed to insufficient precollege preparation. One of the six key points was the idea that because students believed they could be admitted into college regardless of academic performance in high school, they did not take high school seriously and therefore, did not put in the necessary work. In 1964, Stinchcombe pointed out that work-bound students in high school believed high school achievement was not a big deal [20]. The thought was prevalent among college-bound students based on the survey taken at 12 urban and suburban high schools in 1996, where nearly 40% of students who participated in the survey considered exerting effort in high school made only minimal contributions to their future careers [21]. The research done by ACT in 2007 indicated "U.S. primary and secondary-school students spend less time studying than do their counterparts in other industrialized countries but are more satisfied with their academic achievement" [22]. Because of this attitude, some students squandered their time in high school, which resulted in their parents using college savings on remedial courses on subjects which were taught for free in high school. Ultimately, this all contributed to greatly lowering their probability of being awarded bachelor's degrees [19].

## 2.5 Alternative Ways of Predicting

Although it had been employed to predict students' performance in this study, standardized test scores may not be available to analyze in the near future because at least 1,600 colleges changed their standardized test requirement from mandatory to test-optional for the class of 2022 [23]. Alternative ways of predicting students' graduation had to be explored. In other words, other variables that could replace standardized test scores should be quickly identified.

English and mathematics courses comprise the foundation for college study regardless of the college major chosen by students. The grades in English and mathematics courses during the first two college years would predict whether students could succeed in their academic pursuits, and those grades also reflect their precollege preparation [2, 3]. When the students' grades in either English or mathematics were A or B, it could be indicative that the students had a solid foundation of the subjects at the high school level.

Based on *The Toolbox Revisited: Paths to Degree Completion from High School through College* issued by the United States Department of Education [3], if 12th grade high school students in 1992 had taken the following courses counted as Carnegie units, 95% of the students graduated with their bachelor's degrees, and 41% also earned master's degrees, professional doctor degrees, or doctor degrees [3]. The Carnegie unit was named after the American industrialist Andrew Carnegie (1835–1919). One Carnegie unit was defined as 120 hours of contact time with an instructor, such as one hour of instruction a day, five days a week, for 24 weeks, or 7,200 minutes of instructional time over the course of an academic year [24].

- 3.75 or more units English
- 3.75 or more units mathematics
- Highest mathematics of either calculus, precalculus, or trigonometry
- 2.50 or more units of science or two units of core laboratory science, including biology, chemistry, and physics
- Two or more units of foreign languages
- Two or more units of history and social studies
- One or more units of computer science
- More than one Advanced Placement course

This report provided a clearly defined guideline to the professional staff at offices of undergraduate admission to indicate whether the applicants had the potential to graduate if they were admitted.

# 3 Variables and Methodology

## 3.1 Data Used and Dependent Variable

Census day snapshot data of the 2012 freshman cohort at the research university was used to explore and analyze the factors that impacted students' graduation. The 2012 cohort was used in the study with minimal adjustments needed. The dependent variable was whether the students were awarded a bachelor's degree within six years.

## 3.2 Independent Variables

Many factors may ultimately affect students' graduation, such as student demographics, socioeconomic status, pre-college preparation, home location, college major, and the use of learning support [25]. However, not all previously identified factors that may impact graduation were utilized in this study. The rationale of using home location is as a surrogate for family income and parents' education level, which might impact students' learning environment and attitude of studying. However, home location had high levels of missingness and was therefore not employed. As there is some variability by major, major is correlated with on-time graduation. Because the study did not differentiate the graduation rates by specific majors, academic major at time of graduation was not analyzed. As this study was more of a quantitative study, the use of learning support, such as student counselling service, was not selected. Better logic for the exclusion is inability to extrapolate. Effectiveness of learning support likely varies widely. The independent variables employed in the study spanned four separate categories: demographic related information, socioeconomic status, college preparedness, and academic performance, which included five independent variables related to students' first two years' academic performances, especially in English and mathematics subjects.

### *3.2.1 Demographics*

Demographics related variables included students' gender and race/ethnicity. Gender was categorized as male or female. The race/ethnicity classification was also simplified to two groups: underrepresented minority or other. The underrepresented minority group included Nonresident Alien (international students), Hispanic/Latino (their ancestors came from any country which is located beyond the southern boundary of the United States in the Western Hemisphere), American Indian or Alaska Native, Black or African American, Native Hawaiian or Other Pacific Islander, Two or more races, as well as Race and ethnicity unknown. The other group consisted of students whose race was either White or Asian. The definitions of race/ethnicity categories came from the US Census 2010.

### *3.2.2 Socioeconomic Status*

Two independent variables fell under the socioeconomic status category: first-generation students and Pell Grants recipients. If neither parent ever graduated from a four-year college, the student was a first-generation student. Pell Grants were awarded to students from low-income families. Most Pell Grant recipients came from families with annual incomes of less than $20,000, though students from families with an annual income up to $50,000 may still be eligible [26].

### *3.2.3 Standardized Test Scores*

Standardized test score, either SAT or ACT, were used as one of the independent variables related to college preparedness. If students only submitted their ACT scores, the scores were converted to SAT scores according to the Concordance Tables of the College Board.

### *3.2.4 English Preparation*

Since more and more institutions adopted the test-optional policy to admit applicants, an alternative way of testing students' precollege preparation was to track which English and mathematics courses were taken by the students in their freshman and sophomore years at college, and the grades they received from these courses. ENGL 101 and ENGL 102 were the most popular entry-level English courses teaching English composition and were both mandatory for all students, unless waived. Since most students selected one of the two as their first English course, the students had sufficient college preparedness in English. Therefore, there was too little variability to use as a predictor. Thus, this variable was not included in the model.

### *3.2.5 Mathematics Preparation*

Rather than just going by the mathematics courses and grades shown on the students' high school transcripts, this study utilized difficulty of first mathematics course as indicated by the Mathematics Placement Test. Typically, a Mathematics Placement Test was utilized to measure the initial mathematics proficiency of students when they first started college, since each student's mathematics proficiency could widely vary. As Tennant said in 2014, "high school preparation, particularly in mathematics, plays a major role in students earning their bachelor's degree" [27]. Some students who took their first mathematics course in a lower difficulty cluster can later take others in a higher cluster after enough preparation; however, the difficulty level of their first mathematics courses was indicative of students' initial mathematics proficiency upon entering college. Therefore, Mathematics Difficulty, as a variable, could be used to predict the students' graduation success in the model.

The ten most popular mathematics courses first taken by the students can be categorized in four clusters based on the difficulty level. The more difficult a course was rated to be, accordingly, the higher the extent of the student's prior mathematics preparedness. The content of Basic College Mathematics was equivalent to Algebra II from the high school curricula. Precalculus taught students trigonometry and analytic geometry. Usually, these two courses were not even offered in prestigious private colleges, such as the eight Ivy League universities, Massachusetts Institute of Technology, or Stanford University, based on the assumption that these skills have already been learned and mastered. Therefore, these two courses were coded as 1 for the variable Mathematics Difficulty. Introduction to Statistical Reasoning and Elementary Statistics were coded as 2 in Mathematics Difficulty while Finite Mathematics and Elementary Differential Equation were coded as 3. Calculus cluster was broken down into four courses, Calculus for Business Administration and Social Sciences, Calculus I, Calculus II, and Vector Calculus. If students' majors were business, science or engineering related, the students should take at least one course from the calculus cluster. Although there were some differences in difficulty level in the cluster, all four calculus courses were coded as 4 in Mathematics Difficulty. Since this variable, Mathematics Difficulty, was derived from the students' mathematics enrollment records, the distribution of the enrollment from the first mathematics course was presented following

Table 1: Cluster of Mathematics Course and Difficulty

| Cluster of Mathematics Course and Difficulty | | Total Enrollment | |
|---|---|---|---|
| | | # | % |
| 1 | MATH 111---Basic College Mathematics | 1,219 | 29.5 |
| | MATH 115---Precalculus Mathematics | 355 | 8.6 |
| 2 | STAT 110---Introduction Statistical Reasoning | 738 | 17.8 |
| | STAT 201---Elementary Statistics | 156 | 3.8 |
| 3 | MATH 170---Finite Mathematics | 80 | 1.9 |
| | MATH 242---Elem Differential Equation | 8 | 0.2 |
| 4 | MATH 122---Calculus for Business Administration and Social Sciences | 836 | 20.2 |
| | MATH 141---Calculus I | 489 | 11.8 |
| | MATH 142---Calculus II | 155 | 3.7 |
| | MATH 241---Vector Calculus | 99 | 2.4 |
| 10 Major Math Courses | | 4,135 | 100.0 |

### 3.2.6 Repetition of Taking STEM Courses

In previous section, mathematics preparation for college education was addressed. As some students had inadequate mathematics preparation, it not only impacted the students' understanding and digestion of the contents of college level mathematics, but had ripple effect on other subjects, especially sciences. In general, science majors other than biology have been less popular in the United States. At this university, the general education requirements included seven mandatory credit hours of science subjects, in addition to the mathematics/analytic reasoning requirement. Therefore, the challenge of taking the mathematics and science subjects for many students, especially non-STEM majors, was obvious. Because of inadequate preparation in the sciences and mathematics, not all the students who initially chose STEM majors ended up graduating with degrees in those STEM majors. Although many other factors might contribute to their major change, one of the main reasons was that these students could not overcome the challenge of having more rigorous requirements built into the STEM curricula.

After tracking the courses taken by the students, the most popular courses that students repeated to take were six introductory STEM courses after their failing at least one attempt. Based on the tracking, a variable was created to indicate the students' precollege preparation. If students retook any of the six courses, this variable was coded as the cumulative number of times the courses were retaken. The six courses were BIOL 101: Biological Principles I, BIOL 102: Biological Principles II, CHEM 111: General Chemistry I, CHEM 112: General Chemistry II, GEOL 101: Introduction to the Earth, and MATH 141: Calculus I. If students did not retake any of the six courses, it was coded as zero. The observed range of this count variable was 0 to XX. Because the contents of the six STEM courses were introductory level courses of each discipline, it could be used as an indicator of whether students had proficient knowledge and mastery of the same disciplines in high school.

### 3.2.7 Academic Performance

Five variables related to students' academic performance were included as independent variables in the model: the numbers of English and mathematics courses taken separately during first two years whose grades were C or better; total grade points of the English and mathematics courses converted from the grades of the courses separately; their first-year GPA. When a student took same course more than one time, the better grade of the course was applied to calculate the first-year GPA and grade points of both English and mathematics courses. Grade points of English were calculated in following way, a student took both ENGL-101 and ENGL-102 with three credits for each of the course. The grade of ENGL-101 was A (4 points), which was converted to 4 * 3 = 12 grade points; the grade of ENGL-102 was B (3 points), which was converted to 3 * 3 = 9 grade points. Therefore, the total English course points of the students were 19. So did mathematics.

### 3.3 Logistic Regression and Stepwise Regression

Agresti [28] defined logistic regression as

$$\pi(x) = \frac{exp\ (\alpha + \beta x)}{1+exp\ (\alpha+\beta x)} = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

The alternative way of above formular can be expressed as

$$\text{Logit }(P) = \text{Log }(\frac{P}{1-P}) = \alpha + \beta x$$

Agresti [28] said "the random component for the (success, failure) outcomes has a binomial distribution. Whereas π, the probability of the outcome occurring, is restricted to the 0-1 range, the logit can be any real number. The real numbers are also the potential range for the linear predictors (such as α + βx) that form the systematic component of the GLM, so this model does not have the structural problem that the linear probability model has." For the sake of finding the most influential variables in the model, stepwise regression was applied in the data analysis. This method can be used in both linear regression and logistic regression since "stepwise regression provides an important modification of forward selection in that, at each stage of selection, all predictors currently in the model are evaluated [29]."

Dependent variable was the status of graduation. The independent variables were employed to predict students' graduation status. The fitted model should be generated based on the following formula by running the SAS programming.

$$\text{Logit (Prob(graduation}|(x)) = \alpha + \beta_{1\ Sex} X_1 + \beta_{2\ Race} X_2 + \beta_{3\ SAT} X_3 + \beta_{4\ ENGL\ Point} X_4 + \beta_{5\ MATH\ Point} X_5 + \beta_{6\ ENGL\ Count} X_6 + \beta_{7\ MATH\ Count} X_7 + \beta_{8\ GPA\_1ST\_Year} X_8 + \beta_{9\_1ST\_Gen} X_9 + \beta_{10\ Pell} X_{10} + \beta_{11\ 6\_STEM\_Course} X_{11} + \beta_{12\ Math\ Difficulty} X_{12}$$

Gender, Underrepresented minority or not, First-generation students or not, Pell Grants recipients or not, Standardized test scores, Mathematics Difficulty, Retaking the six STEM courses, the numbers of English and mathematics courses taken separately during first two years whose grades were C or better; total grade points of the English and mathematics courses converted from the grades of the courses separately, and the first-year GPA were 12 independent variables employed to predict students' graduation status. These 12 independent variables were tested by logistics regression to generate a fitted model.

# 4 Outcome

## 4.1 Initial Descriptive Data Analysis

As previously discussed, the grade of the first English course, the grade of the first mathematics course, as well as the STEM Courses Retaken were presented below to illustrate the students' proficiencies in English, mathematics, biology, chemistry, and geology when they first came on campus.

### *4.1.1 English Proficiency*

The initial data analysis provided information regarding the students' precollege English preparation based on what they took as their first English courses at the university. The five most popular English courses taken by the students were ENGL 101: Critical Reading and Composition, ENGL 102: Rhetoric and Composition, ENGL 282: Fiction, ENGL 283: The Themes British Writing, and ENGL 285 Themes American Writing. The first two English courses were mandatory to all students as a part of the requirements of general education unless waived. Eighty-three percent and 92.7% of the students got either As or Bs in the first two mandatory English courses, respectively. Students who got either As or Bs in the rest three English courses were respectively 92.1%, 86.7%, and 85.6%. Overall, 88.1% of students got either As or Bs in five of the most popular English courses. Only 6% of students failed their first English course according to the summary of the grades from all five courses, as illustrated in Figure 2. Since all the five most popular English courses were true college-level courses taken first by the students upon arriving on the campus, this demonstrated strong evidence that most students already had adequate English proficiencies. Because the students' English proficiencies might be good enough to handle the challenges of college courses, English preparation was of less concern and therefore, not utilized as a variable in the model to predict students' graduation in this study.

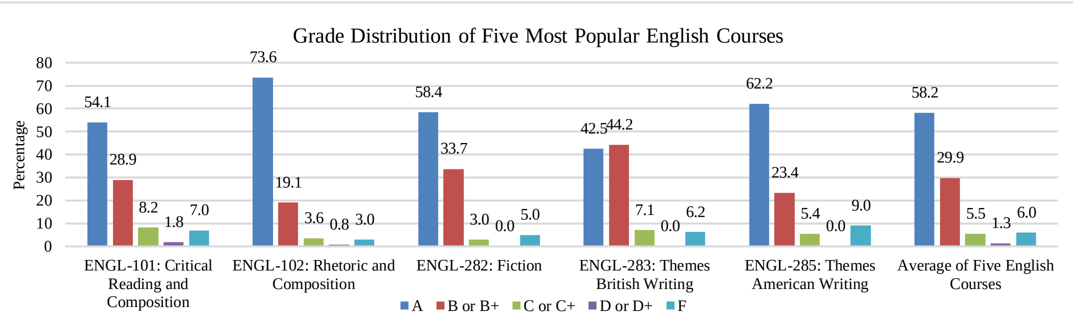


Figure 2: Grade Distribution of Five Most Popular English Courses

### *4.1.2 Mathematics Proficiency*

Based on the initial descriptive data analysis, students' mathematics preparation was less sufficient. More than one third of students needed to take mathematics courses which were composed of curriculum concepts that were already a part of high school mathematics curricula. As Rosenbaum [19] pointed out, the students paid college tuition to essentially relearn the contents of high school classes which had been free. As Table 1 in section 3.2.5 Mathematics Preparation presented, the Mathematics Placement Test results showed that 38.1% of students were advised into either Math 111: Basic College Mathematics (College Algebra/Algebra II) or Math 115: Precalculus. The remaining 21.6% of students took Introduction to Statistics, 2.1% took Finite Mathematics or Elementary Differential Equation, and 38.2% took Calculus. Descriptive statistics above were summarized based on the ten most popular mathematics courses first taken by the students. The above Table 1 supplied more detail information of the first mathematics course taken by the students. Certainly, according to Analytical Reasoning and Problem Solving (ARP) general education requirement of this university, students in non-STEM majors may take entry level computer sciences or philosophy courses to fulfill the ARP requirements in order to graduate.

In Figure 3, the order of the five mathematics or statistics courses first taken are presented by their popularity. In Math 111: Basic College Mathematics 58.5% of students got As or Bs as compared to 54.1% of students who received the same grades in Math 115: Precalculus. In comparison to the percentages of students receiving As or Bs in the two mandatory English composition courses, the percentages of students getting A or B in the two mathematics courses were notably lower. Though the content of these two mathematics courses was at the high school level, most students could not achieve the high grades as they received in English courses. Across the five mathematics courses most often taken by the students, 12.2% of students got Fs. The descriptive statistics discussed above showed that the students faced more difficulties in mathematics and reconfirmed Adelman's conclusion from his 1999 report in which he highlighted the crucial role of mastering mathematics during the high school years [2].

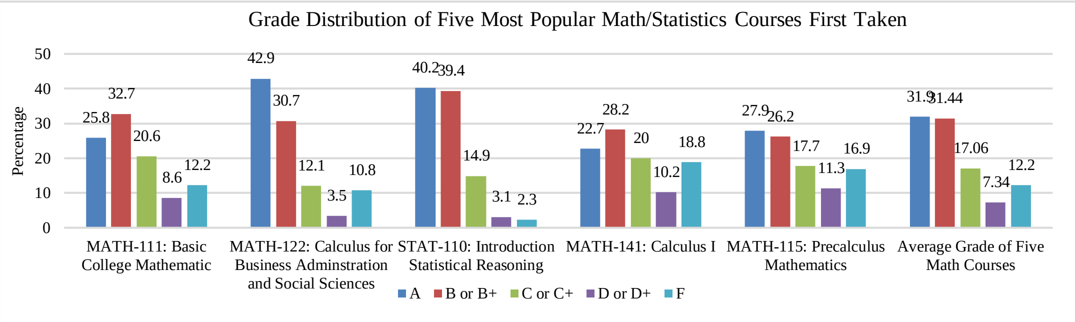


Figure 3: Grade Distribution of Five Most Popular Mathematics/Statistics Courses

Of the five most popular mathematics courses taken by the students, MATH 141: Calculus I had the lowest passing rate. The course had 2,311 grades recorded because some students took the course multiple times. When the 2,311 grades were separated by first calculus attempt group, who took MATH 141 as their first mathematics course, versus subsequent attempt group, who had taken Math 111: Basic College Mathematics and/or Math 115: Precalculus first then took MATH 141, the overall grade distribution revealed stark disparities. For the sake of fair comparison, only first attempt grades were used in the data analysis, ignoring succeeding attempts after first fail. Compared to all the officially recorded grades of the two groups, first calculus attempt versus subsequent attempt group, 22.7% of the first calculus attempt group got As with only 13.9% students from the subsequent attempt group got the As grade from their first attempting; 28.2% of students received a B or B plus vs. 21.1%; 20.0% with a C or C plus vs. 19.4%; 10.2% with a D or D plus vs. 12.7%; and finally, 18.8% with a F vs. 32.9%. The students with a C or better grade for the first calculus attempt group was 70.9%, while the same rate in the subsequent attempt group was only 54.4%. Although not all students were required to take MATH 141: Calculus I since it targeted students in STEM majors, the higher failure rate of MATH 141: Calculus I mirrored the students' mathematics preparation that were taught during their high school years. As not all students had adequate mathematics preparation to face the challenges of mathematics courses in college, many colleges, including the university which the study targeted, offered sort of remedial mathematics courses, such as Math 111: Basic College Mathematics and Math 115: Precalculus offered at the university. The results of this study indicated following two points. First, if students had not laid a solid foundation of mathematics from high school, it was difficult for the students to understand and apply what they learned from mathematics courses which were prerequisites of calculus just by taking Math 111: Basic College Mathematics and/or Math 115: Precalculus. Secondly, it explained why some students in STEM majors either transferred to non-STEM majors or dropped out because of their insufficient mathematics precollege preparation. Simply put, they could not overcome the obstacles they faced when taking college level mathematics courses and other STEM courses which had also mathematics as their foundation.

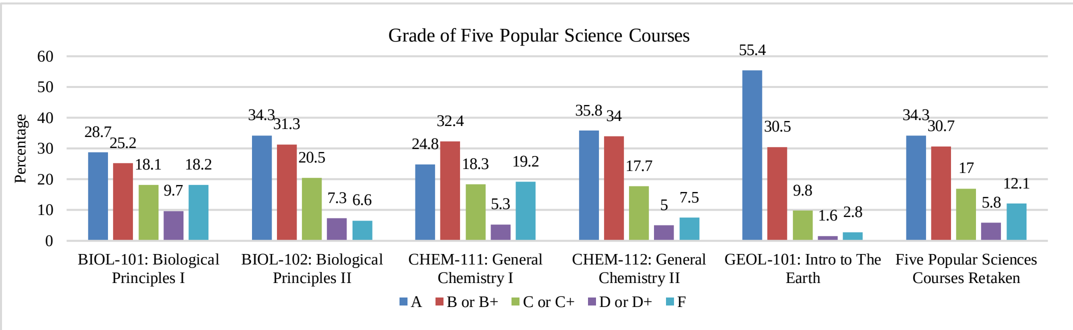


Figure 4: Grade Distribution of Five Popular Science Courses

## 4.2 Observations Used and Fitted Model

### *4.2.1 Observations Used*

Although 4,578 students were in the cohort, only 3,226 students whose data were complete for all 12 independent variables were included in the model. After running a stepwise logistic regression model in SAS, six independent variables were selected and found to be significant.

Before exploring the roles of the six independent variables in the fitted model, it was necessary to find out who were excluded from the model as missing values. The reasons, why the observations were deleted in the fitted model as missing values, were that the students either did not take any English/Mathematics courses in their first two-year college study, or all grades of the courses of the two subjects taken by them were below C that cannot be counted. As the students in the missingness group had either English or mathematics credits from Advanced Placement (AP) test scores, or transferred credits, or dual enrollment credits while they were high school students, the general education requirements of taking courses from the two subjects were waived. This was primary reasons why 1,352 students became missingness, of which the reasons were counted 97.26% missingness. The rest missing observations came from the students who missed either first-year GPA, or Standardized test scores, or Mathematics Difficulty; as well as dropped out the college during any semester of the two years. Since Gender, Underrepresented minority or not, First-generation students or not, Pell Grants recipients or not, Retaking the six STEM courses were either cleaned or redefined before the data analysis, the five variables did not contribute to any missingness to the model.

If students had missing values of Numbers of English and mathematics courses taken, logically, they must miss the values of Final grades earned in English and mathematics course. Because of AP credits honored or other transfer credits, some students in the missingness group did not missing values of Final grades earned in English and mathematics course though they did have missing values of Numbers of English and mathematics courses taken. Therefore, the missing values of Final grades earned in English and mathematics course were subgroups of missing values of Numbers of English and mathematics courses taken separately.

Since missingness in the data analysis may lead to bias conclusions, the issue should be addressed. One solution is to assign reasonable values to the variables which cause missingness. The comparison between the model omitting the missingness and the model including the missingness whose missing values were assigned would be addressed and discussed in section 4.6 Comparison of Fitted Model and Model Including Missingness.

### *4.2.2 The Fitted Model*

The stepwise logistics data analysis selected the six variables in the fitted model. Among the six variables, first-year GPA made the largest contribution to the students' graduation; Numbers of English and mathematics courses taken, as well as Mathematics Difficulty also had positive impact to the students' graduation. In other words, the more English or Mathematics courses taken, or the more advancer level of first mathematics course taken, the higher possibility of graduation. First-generation students or retaking the six STEM courses had negative impact to students' graduation.

Logit (Prob(graduation|(x)) = -3.6634 +0.2662*ENGL_Count

+ 0.4975*MATH_Count + 1.0164*GPA_$1^{ST}$_Year – 0.4931*_$1^{ST}$_Gen

- 0.1545*6_STEM_Course + 0.1676*Math_Difficulty

### *4.2.3 Descriptive Statistics of Variables in the Fitted Model*

The average first-year GPA was 3.285, and the average counts of English and mathematics courses taken by the end of sophomore year were 2.232 and 2.462, respectively. Three hundred seventy-five (11.75%) students were first-generation college students in this study.

Since 1,630 (50.53%) students did not retake any of the six STEM courses, the average count of students retaking the six STEM courses was 1.0567. Among the 3,226 students in the fitted model, 1,235 (38.3%) students took at least either Basic College Mathematics or Precalculus, 713 (22.1%) students took Introduction Statistical Reasoning and/or Elementary Statistics, 68 (2.1%) students took Finite Mathematics or Elementary Differential Equation, and 1,210 (37.5%) students selected Calculus as their first mathematics course.

## 4.3 Hypothesis Tested for the Fitted Model

The Global Null Hypothesis: $\beta = 0$ was tested. This logit model was statistically significant. The reported likelihood-ratio (LR) tests that GRADUATION was jointly independent of the predictors simultaneously; $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$. The LR test statistic of 415.9301 was chi-squared ($\chi 2$) with six degrees of freedom and *p*-value < 0.0001. In addition, all predictors of GRADUATION were statistically significant as their *p*-values were less than 0.05. Hosmer and Lemeshow Goodness-of-Fit Test also provided additional evidence the model fits the data well ($\chi 2$ = 15.3614; *df* = 8; *p* = 0.0692), since "the Hosmer-Lemeshow statistic indicates a poor fit if the significance value is less than 0.05" [30].

The below Table 2 was the Maximum Likelihood Estimates of the six predicators which was selected in the fitted model. It meant that the six variables were statistically significant as their p-values were less than 0.05.

Table 2: Analysis of Maximum Likelihood Estimates of the Six Predicators

| Analysis of Maximum Likelihood Estimates | | | | | |
|---|---|---|---|---|---|
| Parameter | DF | Estimate | Standard Error | Wald Chi-Square | Pr > ChiSq |
| Intercept | 1 | -3.6634 | 0.3061 | 143.2264 | <.0001 |
| 1st_Generation | 1 | -0.4931 | 0.1343 | 13.4794 | 0.0002 |
| ENGL_Count | 1 | 0.2662 | 0.0467 | 32.4333 | <.0001 |
| MATH_Count | 1 | 0.4975 | 0.0501 | 98.4842 | <.0001 |
| GPA_1ST_Year | 1 | 1.0164 | 0.0815 | 155.6505 | <.0001 |
| 6_STEM_Courses | 1 | -0.1545 | 0.0373 | 17.1449 | <.0001 |
| Math_Difficulty | 1 | 0.1676 | 0.0394 | 18.1000 | <.0001 |

## 4.4 Multicollinearity

"Multicollinearity can be briefly described as the phenomenon in which two or more identified predictor variables in a multiple regression model are highly correlated. The presence of this phenomenon can have a negative impact on the analysis as a whole and can severely limit the conclusions of the research study" [31].

As this was categorical data analysis, Spearman Correlation Coefficients can be used to detect if the model was multicollinearity between dependent variable and independent variables because it is based on the ranked values for each variable rather than the raw data [32, 33]. The next step was to find out particularly high correlations between dependent variable and any independent variables in the model by using Spearman Correlation Coefficients. "A coefficient of correlation of +0.8 or -0.8 indicates a strong correlation between the independent variable and the dependent variable" [34]. The strongest correlation between GPA_1ST_YEAR, one of the 12 independent variables in the model, and Graduation, the dependent variable, was found to be 0.33591. Since that value was much smaller than 0.8, there was no multicollinearity in the model.

## 4.5 The Fitted Model Applied

Six variables were selected in the fitted model by the stepwise logistic regression. Positive values of the coefficients of the four variables, which included first-year GPA, counts of English and mathematics courses taken, as well as difficulty of the first mathematics course taken, indicated larger positive effects on graduation. However, since both coefficients of first-generation and repeated taking six STEM courses were negative values, they affected students' graduation negatively. The impacts on graduation of the first mathematics course taken and repeated taking six STEM courses could be estimated by applying the fitted model for 12 students. Since Math Difficulty had four different subgroups and Repeated Taking Six STEM Courses was separated by taking zero, one, and two of the STEM courses, 12 students were used to estimate the model. As shown in 4.5.1 to 4.5.4, the estimations were based on the number of repeated taking six STEM courses and which mathematics courses they had first taken, either Basic College Mathematics or Precalculus (the first subgroup), Introduction to Statistics (the second subgroup), Finite Mathematics or Elementary Differential Equation (the third subgroup), and Calculus (the fourth subgroup). In this way, there were three students in each subgroup for estimating. Furthermore, based on the value of the variable that students repeated taking six STEM courses, one student was assigned as never having to retake any of the six STEM courses, another student retook one, and last one retook two. Although the two variables mentioned above varied by each estimation, the other four variables from the fitted model were constant. The 12 students were not first-generation, and all took two mathematics and two English courses respectively. The students were assigned first-year GPA as 3.285, which was average of the students in the fitted model.

### *4.5.1 The First Subgroup*

Three students were assigned to take MATH-111: Basic College Mathematics or MATH-115: Precalculus (Math Difficulty 1, see Figure 5) based on their scores of the Mathematics Placement Test. If the first student in the group, as seen on the top line in Figure 5, did not need to retake any of the STEM courses, the graduation rate of the student would be 79.7%, which was higher than the overall observed graduation rate of 77.0% for this cohort. When the second student in the group retook one of the STEM courses, as shown in the middle line of Figure 5, the graduation rate dropped to 77.1%. When the third student in the group as seen in the bottom line of Figure 5 retook two of the STEM courses, the graduation rate dropped again to 74.3%. The interpretation of the lines for other three subgroups were as same as the first subgroup described above.

### *4.5.2 The Second Subgroup*

Three students were assigned to take MATH-110: Introduction Statistical Reasoning or MATH-201: Elementary Statistics (Math Difficulty 2, also see Figure 5) based on their scores of the Mathematics Placement Test and major. If the first student in the group did not need to retake any of the STEM courses, the graduation rate of the student would be 82.3%. When the second student in the group retook one of the STEM courses, the graduation rate would be 80.0%. When the third student in the group retook two of the STEM courses, the graduation rate would be 77.4%.

### *4.5.3 The Third Subgroup*

Three students were assigned to take MATH-170: Finite Mathematics or MATH-242: Elementary Differential Equation (Math Difficulty 3, also see Figure 5) based on their scores of the Mathematics Placement Test and major. If the first student in the group did not need to retake any of the STEM courses, the graduation rate of the student would be 84.7%. When the second student in the group retook one of the STEM courses, the graduation rate would be 82.5%. When the third student in the group retook two of the STEM courses, the graduation rate would be 80.2%.

### *4.5.4 The Fourth Subgroup*

Three students were assigned to take MATH-122: Calculus for Business Administration and Social Sciences or MATH-141: Calculus I or MATH-142: Calculus II or MATH-241: Vector Calculus (Math Difficulty 4, also see Figure 5) based on their scores of the Mathematics Placement Test and major. If the first student in the group did not need to retake any of the STEM courses, the graduation rate of the student would be 86.7%. When the second student in the group retook one of the STEM courses, the graduation rate would be 84.8%. When the third student in the group retook two of the STEM courses, the graduation rate would be 82.7%.

According to the discussion above, both the difficulty of the first mathematics courses taken and the number of times the six STEM courses were retaken had great impacts on students' graduation. Because there were overall lower mathematics grades compared with English course grades and nearly half the students retook at least one of the six STEM courses after failing their first attempt, students' first-year GPAs were negatively affected, which ultimately impacted graduation.

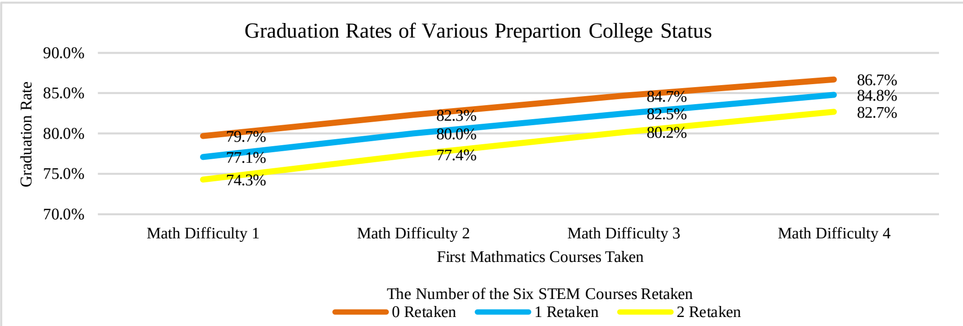


Figure 5: Graduation Rate of Various Preparation College

## 4.6 Comparison of Fitted Model and Model Including Missingness

The observed graduation rate of cohort 2012 with 4,578 observations was 77.0% compared to 79.9% with 3,226 observations from fitted model. The graduation rate of 1,352 missingness was 70.12%. Since Numbers of English and mathematics courses taken, the first-year GPA, and Mathematics Difficulty were factors contributed positive effects to students' graduation in the fitted model, any missing values from one of the four variables would obviously drag down the graduation rate. This explained the lowest graduation rate of missingness compared to the rates not only fitted model but observed cohort 2012.

The missing values of seven variables, which included Numbers of English and mathematics courses taken, English Point and Mathematics Point (grades earned in English and mathematics courses), first-year GPA, SAT-2005, as well as Mathematics Difficulty, were added by using means of the variables in the fitted model discussed in previous sections. In this way, all students in the cohort 2012 had the values of the 12 independent variables that were used to predicate student graduation. After running SAS logistic regression, exactly same six variables were selected as the model including 1,352 missingness.

Among all predictors of GRADUATION, the six variables discussed above, which missing values of each variable were assigned for the sake of fixing missingness, were statistically significant as their p-values were less than 0.05. Hosmer and Lemeshow Goodness-of-Fit Test provided additional evidence the model fits the data well ($\chi 2 = 14.5198$; df = 8; p = 0.0676). Therefore, it can be more confident that the results of the fitted model were generalizable to all students in the cohort though there were 1,352 missingness.

# 5 Discussion

As discussed in 4.1.1 English Proficiency, English proficiencies of the students were not an issue. Thus, the following discussion focused on mathematics proficiency, which had been one of the primary graduation blockers. The path to gaining mathematics proficiency could be traced back to the students' kindergarten to 12th grade (K-12) educational journey. Thus, it can be extrapolated that the correlation between precollege preparation and college graduation can also be seen as the specific correlation between precollege mathematics proficiency and college graduation.

## 5.1 Weaker Predictor by Using SAT or ACT Test

In the cohort, 2,888 students submitted their SAT test scores, of which the $50^{th}$ percentile of SAT Verb was 590 and SAT Mathematics 608. On the other hand, 2,779 students submitted their ACT test scores, of which the $50^{th}$ percentile of ACT English was 25 and ACT Mathematics 25. Some students submitted both SAT and ACT test scores. The $50^{th}$ percentiles of English/mathematics scores were similar or same in both tests. But why did the mathematics proficiencies of the students become a problem and not English? As illustrated in 4.1.1 English Proficiency, students' English was good enough to cope with the challenge of college English courses, but not mathematics courses since both SAT and ACT mathematics sections only tested the contents of middle school or even elementary school level mathematics, such as dividing fractions. Thus, neither SAT nor ACT mathematics sections were discriminating enough to measure the students' mathematics proficiency required in college. This was the reason why freshmen had to take the Mathematics Placement Test after they came to campus even when standardized test scores were mandatory. Therefore, compared with the other six selected variables, standardized test scores were not a significant factor to predict the students' graduation.

## 5.2 Philosophy of Contents of Mathematics Taught in Secondary Education

Around 100 years ago, two schools of thought on American mathematics education had debated on appropriate curriculum in secondary education. In fact, both schools of thought came from professors at the Teacher College at Columbia University. One of them was E.D. Hirch, Kilpatrick, whom Dr. John Dewey guided. Kilpatrick's primary point was that neither algebra nor geometry should be taught to students in secondary education unless the contents were proven to be useful. He did not believe that mathematics contributed to mental discipline. His thoughts were reflected in the report titled *The Problem of Mathematics in Secondary Education* published by the U.S. Commission of Education in 1920 [35]. Another school of thought was represented by David Eugene Smith who objected to Kilpatrick's opinion and published another report titled *The Reorganization of Mathematics for Secondary Education* in 1923 [36]. This later report exerted some influence on public education, like the College Examination Board, the former name of the College Board, which included both mathematics and English verbal in the earliest version of SAT in 1926. Specifically, the 1926 version of SAT contained nine sub-tests; two sub-tests of mathematics which included arithmetical problems and number series, and seven sub-tests of verbal skills comprised of definitions, classification, artificial language, antonyms, analogies, logical inference, and paragraph reading. Compared with the earlier method initiated in 1901, which involved an agreement of the presidents of 12 private universities in the northeastern United States to use an essay to test their applicants by the College Examination Board, the SAT in 1926 was more comprehensive [37]. However, the impact of that change was still very small. The earlier report by Kilpatrick still had more influence on secondary education than the latter report until the former Soviet Union (U.S.S.R) launched Sputnik I on October 4, 1957 [35]. One example was that between 1933 and 1954, the number of students who took geometry decreased even though overall enrollment in high school soared [38]. Although launching Sputnik I by the former U.S.S.R spurred people to give more attention to mathematics and science education, the debate regarding the composition of secondary education's curriculum never ended. On one hand, more students took algebra, geometry, trigonometry, and even calculus during their secondary education period. Most of the states created minimum competency tests to measure students' basic mathematics skill since the mid-1970's [35]. On the other hand, the phenomenon described by Adelman's reports [2, 3] continued to be a serious issue faced by students, as analyzed, and demonstrated in this study.

## 5.3 Mathematics Proficiency

Because calculus requires a more solid foundation of function, logarithm, exponent, trigonometry, and analytic geometry, it is likely that only students in the STEM or business majors would elect to take one of the calculus cluster courses. Since students who choose to take calculus later may do so because they lack the necessary academic and psychological preparation to take it initially, it was unsurprising that the rate of students failing MATH-141: Calculus was 32.9% for students who chose to take the course as their non-first mathematics course. In contrast, the fail rate was only 18.8% for students who chose to take it as their first mathematics course, indicating that these students possessed a higher level of preparation.

The following science courses are ranked from having the least to the most mathematics prerequisites: geology, biology, chemistry, and lastly, physics. Since the students had to earn seven credits from the science disciplines to fulfill their general education requirements, they selected the least mathematics-forward course, geology, which resulted in a higher pass rate. On the flip side, physics was not even included as part of the six STEM courses as most students opted not to take it since it included the most rigorous mathematics prerequisites. For example, one such mathematics prerequisite was Calculus I which was rated a level 4 Mathematics Difficulty course. General Chemistry I had a lower pass rate but was the most popular of the five sciences courses discussed earlier. While not as difficult as physics in terms of mathematics prerequisites, General Chemistry I at least requires knowledge of Algebra I, which is typically learned in middle school. For instance, problems related to making a solution with a desired concentration is fundamentally based on the concept of ratios and solving for an unknown variable when all other variables are known (e.g., how many mL of 5 M NaCl solution should be added to a 100 mL solution of 1 M NaCl solution to obtain a solution having NaCl concentration of 2 M). Unfortunately, many students may not have retained such knowledge by the time they enter college, which could have caused a lower pass rate in the five sciences courses.

## 5.4 Why is Mathematics So Challenging?

*Mathematics' Double Standard*, published by Achieve, Inc [39] in March 2013 indicated that "Far too many students in the U.S. give up on mathematics early because it does not come easy and they believe only students with innate ability can really be 'good' at mathematics, a notion that is all too often reinforced by adults who believe the same thing. There is a serious gap between how American people value mathematics generally and how they value mathematics for their own enrichment." Since mathematics skills are foundational for later learned skills, these types of thoughts and subsequent behaviors based on the thoughts result in a domino effect where students are getting further and further behind, to the point where it is almost impossible to catch up. These misconceptions almost certainly must change to reverse the current trend.

Additionally, schools' curricula and textbooks should also be addressed. Gorman [40] believed that American mathematics instruction often relied on rote memorization rather than a true, deeper mastery of many mathematics concepts. Unlike Singapore's mathematics curricula, American mathematics textbooks cover very broad topics, and similar topics may be too repetitive over several grades while others are barely touched upon. Without a thorough understanding and extensive practice, the concepts and skills learned were forgotten very quickly, especially because most students attended public schools where students did not keep their textbooks each year. With the limited time of the school year, it was difficult to retain all mathematics knowledge learned that year without having their study materials to refer to. In the absence of a rigorous and well-designed mathematics curricula and academic support system, it was not surprising that many American students struggled in mathematics and did not prepare themselves well with the foundational mathematics skills that were required for the success in their college education.

## 5.5 Assessment of Mathematics Proficiency Globally

In previous sections, philosophies, attitudes, thought processes, and behaviors of the public towards mathematics in secondary education were discussed. Differences in pedagogy and curricula of teaching mathematics between American and East Asian Cultural Sphere countries, utilizing Singapore as an example, were addressed. Because of these reasons, when the Program for International Student Assessment (PISA) released their test results, the rankings of the United States were not surprising to educators and educational researchers in America. OECD managed the Global Rankings on Student Performance in Mathematics, Reading, and Science every three years for 15-year-old students. To evaluate educational systems, PISA involved more than 90 countries and economies with approximately three million students worldwide since 2000. See the results of the 2018 top nine ranking countries and economies [41], as well as the United States in Figure 6.

Compared with the rankings in 2015, the 2018 rankings of the United States saw some improvements. The ranking of reading rose to 13 from 24, mathematics 37 from 39, and science 18 from 25. In these three assessment areas, English was the strongest, and mathematics was still the weakest. As the rankings were consistent year by year, it truly reflected proficiencies of American students in these three areas. Students who eventually went onto college were a subgroup of the overall student body in high school, and some high school students represented the United States in the sample group to participate in the PISA tests.

Therefore, it was not out of the ordinary to see that most college-bound American students had sufficient English proficiency at four-year college but faced all kinds of challenges in mathematics when they began their study in college.

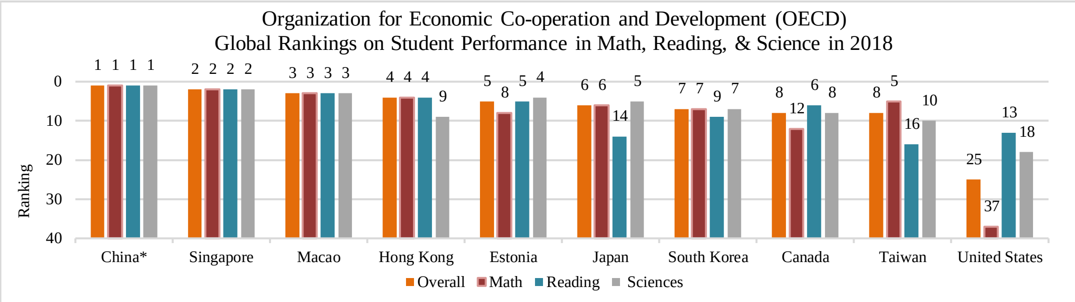


Figure 6: Global Rankings on Student Performance in Math, Reading, & Science in 2018

*Students from Shanghai, Beijing, Jiangsu, and Zhejiang, which were four provincial level administrative units among 31 in mainland China, participated in PISA 2018.

Certainly, people might have different opinions toward the PISA test, especially since seven of the top nine countries or economies in the 2018 rankings came from East Asia. In the East Asian Cultural Sphere, most students did more schoolwork in terms of recitations and extensive exercises and had more discipline overall. Many students attended afterschool tutoring programs based on the traditional learning methods instead of focusing on encouraging students' creativity, freedom, athletic activities, and leisure time. Since the end of World War II, Japan, Taiwan, South Korea, and Singapore have shown an increasing tendency to integrate the two approaches. Although each region's pursuits mirrored its own philosophy, culture, and rationale, students should still have basic proficiencies of language, which is English in the United States, and mathematics regardless. Even if students' English proficiency showed more optimistic results, both previous reports completed by Adelman and this study unfortunately concluded that there was still a long way ahead to achieve the target in mathematics.

# 6 Conclusion

## 6.1 Factors of Impacting Students' Graduation

As discussed in the 4.5 The Fitted Model Applied, first-year GPA, the count of English and mathematics courses taken in the first two years, difficulty of the first mathematics course taken, whether any of the six STEM courses had to be retaken, as well as first-generation student status, were six variables that had impacts on students' graduation.

## 6.2 Precollege Preparation

The outcome of this study highlighted the importance of precollege preparation in secondary education, especially in mathematics and the science disciplines since most STEM courses were sequential. Contents of the courses in college are built upon the foundation of knowledge and skills learned in secondary education. The weaker spots there were in the foundation along the way, the more of an uphill struggle it would be for students to successfully navigate the way to their educational finish line, which for many students ends at the bachelor's level. With all these various obstacles, it is not difficult to understand why coming to college armed with a strong academic background where students already have a deep understanding of mathematics and science subjects and from many different angles can seriously boost graduation success down the line.

### 6.3 Adelman's Conclusion Confirmed

It was over two decades ago when Adelman issued his first report [2], and fifteen years since his revisited report was published [3]. However, the issues that he had emphasized were still significant for students, especially students' mathematics precollege preparation. The more difficult the first mathematics course taken by the students at college, the higher probability the students could successfully graduate.

The significance of this study and model used emphasized the impact and effectiveness of mathematics precollege preparation, which highlighted the important role of secondary education. Therefore, for the sake of improving college graduation rate, there should be more concentrated efforts focused on laying a stronger foundation for eventual college study before students entered college.

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