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A Mathematical Theory to Evaluate Disruptive Changes Deductively

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Abstract

Digital transformation, known as DX, is gaining a lot of attention in a variety of organizations, and higher education institutions are experiencing such rapid changes. IR sections in them, therefore, have to support and evaluate their trials for such rapid changes. Although a popular approach to evaluating trials is to use data about them, in the case of trials aiming for disruptive changes, it is important to evaluate them beforehand since such a trial can be too costly and its impact can be serious damage to institutions. However, it is intrinsically difficult to deal with such changes since we need not have enough data beforehand. This paper is devoted to developing a framework, which can be used to evaluate such disruptive changes. The main idea for the framework is that our daily behaviors are defined as a *theory* from the perspective of information dissemination. Using the proposed framework, we derive some findings deductively, which are not obvious from existing approaches. Thus we can conclude that the proposed framework is *fruitful*. These findings include that information dissemination is derived to create disruptive changes, although our basic notions do not include being disruptive. We can also show that dissemination can cause a successive cascade and the proposed model can explain the reason why resistance to new changes occurs. These findings can not be obtained from existing defitions for disruptive changes, such as DX. The main contribution of this paper is to show a deductive approach, which is not popular in IR, is effective in evaluating such disruptive changes.

Keywords: Deductive Approach, Fruitful, Mathematical Theory, Dual Process Theory, Information Dissemination

1 Introduction

We consider disruptive changes, such as digital transformation known as DX, in higher education institutions, and how to evaluate trials for such changes. Due to the rapid progress of ICT, our society is unprecedentedly changing. Therefore, higher education institutions must also change themselves. When a higher education institution introduces such changes into its organization, it is necessary for an IR section to support and evaluate it.

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It is, however, essentially difficult to evaluate such changes because they would have a big impact on institutions and thus they could change the goal, which is important when evaluating, of institutions. When we evaluate something, we use some assumptions, whether consciously or not. To evaluate some achievements, for example, heavily depend on the goal of an institution, which plays a role as an assumption. But, disruptive changes may alter assumptions themselves, including organizational goals. Thus we can not use existing assumptions for evaluation in the case of disruptive changes.

In addition to that, the following reason raises the difficulty of evaluating disruptive changes. Because big, disruptive changes can be too costly for institutions and their impact can be serious damage, we want to evaluate their process *beforehand*. However, we need some information, including some results, about the process in order to evaluate it. Therefore, it is essentially difficult if we use inductive[1] approaches to evaluate disruptive changes beforehand. Standard approaches used in IR are inductive, which requires some data about evaluation.

This paper is devoted to developing a theoretical framework, which is used to evaluate a process of disruptive changes. Here, the term "framework" means a model and an evaluation approach, and we use a model or theory interchangeably. After introducing the proposed model, we evaluate it in a deductive approach, where a model is first assumed and individual consequences are derived from the model.

Since we want to treat disruptive changes based on information dissemination, firstly we define information dissemination. When we define something, it is essential to use notions that are already defined or simply understood because we have to evaluate definitions in a step-by-step manner from the used notions. Conversely, if we use undefined terms, we can not evaluate the definition.

After defining the information dissemination, we introduce a *theory*, which is defined as a pair of a set of premises, including definitions, and one of the theorems derived from premises. With this notion, we treat our daily behaviors as theorems of a theory. In other words, our daily behaviors are treated mathematically in the proposed model. Therefore, we can logically derive some findings about our daily behaviors. Based on these notions, we can describe a disruptive change as a new theory with a different premise because there is a logical gap between existing theories and the new one. It is worth noting that we do not use the term "disruptive" in our definitions.

Our main contribution is the notions we define. In addition, another important contribution of this paper is to use of a deductive approach for organizational processes in higher education institutions. The opposite way of it is an inductive way, in which a hypothesis is derived from observations. This means we need to evaluate inductively *after* the process to try to change your institution since we need data from the process. On the other hand, when we evaluate the process deductively, we do not need observations since we use logical derivation in a deductive approach. This enables us to deductively evaluate disruptive changes, which is difficult to evaluate using inductive ways.

It is noteworthy that we use this approach to evaluate the proposed model it self. That is, the introduced model corresponds to an assumed model in a deductive approach, and we derive some consequences from the model. If we obtain useful consequences from an assumed model, then the model is said to be *fruitful*, and we conclude that the model is valid. In our case, we treat our definitions as a model and thus our definitions are valid since we derive useful consequences from them.

[1] Here we assume two types of inference, deduction and induction.

This paper is a value added version of a conference paper published in [1], where mainly figures and examples are added.

2 Related Work

In this section, we briefly review related work on disruptive changes. However, DX or similar disruptive changes have not been well considered in IR, except in some papers, such as [2]. So we first take a look at definitions of *digital transformation (DX)* since DS has drawn a lot of attention to higher education institutions. And then we consider innovation, which also has a meaning of disruptive changes.

The term "digital transformation" was originally introduced in [3]. However, DX was not clearly defined in this paper and thus we can not use the notion of DX deductively. Therefore, some researchers have tried to define it. Along with this line, Vial derived the concept of "DX" inductively from about 300 papers about DX as follows [4]:

> a process where *digital technologies* create *disruptions* triggering *strategic responses* from organizations that seek to alter their *value creation paths* while managing the *structural changes* and *organizational barriers* that affect the *positive* and *negative* outcomes of this process.

When we use this definition in a deductive approach, we need these notions in this definition to be defined clearly. However, some terms, like "strategic" or "value", in the statement are vague. In addition, the term "disruption" is explicitly included in the definition, meaning it is a tautology.

In addition to DX, "innovation" also involves a meaning of similar disruptive changes. For example, non-continuous paths between existing technologies and those created by in-

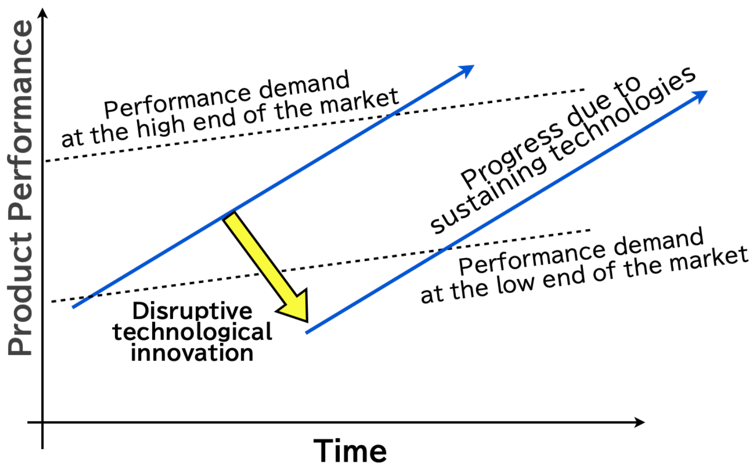

Figure 1: Created based on Figure 1.1 in [5]

novations are clearly depicted in [5] (see Figure 1). We find that usual improvements in technologies constitute a continuous path whereas innovation creates a gap between continuous path. We want to emphasize that some gap is important.

In the first place, IR was defined as follows [6]:

> Institutional research is research conducted within an institution of higher education to provide information which supports institutional planning, policy formation, and decision making

So many researches in this field basically deal with data in institutions and analyze it. However, when we consider disruptions, there does not exist enough information available for IR in general. So we need methods available to evaluate and analyze before a process aiming for disruptive changes starts. We try to overcome this problem with a deductive approach, which enables us to evaluate without data.

3 Preliminaries

Before we propose our model and framework, we introduce some basic concepts for them.

We begin with basic notions in psychology, *System 1* and *System 2*. A dual process theory explains that a human being has two different thinking systems, System 1 and System 2. Kahneman [7] said that "System 1 operates automatically and quickly, with little or no effort and no sense of voluntary control" whereas "System 2 allocates attention to the effortful mental activities that demand it." For example, when we can detect the direction of a sound suddenly heard, the recognition is carried out by System 1. On the other hand, when we count how many times the letter "a" appears on a given page, we need careful attention and thus we use System 2 in this case.

Next, we take a close look at the origin of "information" since we want to consider information dissemination later. According to the Oxford English Dictionary, around the mid-14th century, "information" had the meaning of shaping the mind. Similarly, the New Oxford American Dictionary says that the origin of the term is the formation of the mind or teaching. From these entries in two major dictionaries, the origin of information is to give an explicit form to some part of our mind.

Combining the notions of System 1 and 2, and the origin of information, we can think that to inform is to give a form to something recognized by System 1 so that we can recog-

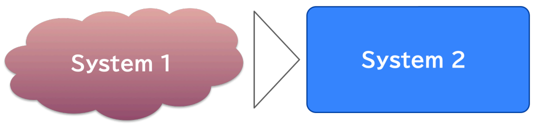

Figure 2: illustration of our image of "inform" using something recognized by System 1 and System 2 [1]

nize the transformed one by System 2.

Finally, we define a *theory* to be a set of *premises* and a set of *theorems*, where premises includes axioms, assumptions, and definitions, and a theorem is derived from premises.

This definition of theories is the same as those in mathematics. If the premises of some theory we defined are true, then all theorems proved from the premises are always true, in contrast to statistical statements, which we often see in many fields of natural science.

As an example of a theory, we show an example of a formal language, called an elementary formal system [8][9].

Example 1 *Let $\{a,b,c\}$ be an alphabet, that is, we consider strings of "a", "b", and "c". For example, "a" and "bbaccba" is a string.*

The following set of equations are axioms, that is, they are premises, where x, y, z are variables.

$$\left\{\begin{array}{r} p(xyz) \leftarrow q(x,y,z) \\ q(ax,by,cz) \leftarrow q(x,y,z) \\ q(a,b,c) \leftarrow \end{array}\right\} \begin{array}{r}(1)\\(2)\\(3)\end{array}$$

We can substitute a string to a variable. For example, we obtain the formula $p(abc) \leftarrow q(a,b,c)$ by substituting a,b,c to x,y,z of (1), respectively.

A substituted term, such as $p(abc)$ and $q(a,b,c)$, is said to be proved if it is equal to some axiom, or equal to the left-hand side of a substituted axiom and the right-hand side is already proved. For example, $q(a,b,c) \leftarrow$ or equally $q(a,b,c)$ is proved because it is equal to (3). Similarly, $q(aa,bb,cc)$ is also proved because $q(aa,bb,cc) \leftarrow q(a,b,c)$ is a substituted axiom and $q(a,b,c)$ is already proved.

A string w is said to be a theorem if $p(w)$ is proved. For example, aabbcc is a theorem because of the following substituted formulae:

$$q(a,b,c) \leftarrow \qquad (\because (3)) \qquad (4)$$

$$q(aa,bb,cc) \leftarrow q(a,b,c) \qquad (\because (2)) \qquad (5)$$

$$p(aabbcc) \leftarrow q(aa,bb,cc) \qquad (\because (1)) \qquad (6)$$

Therefore $p(aabbcc)$ is proved and thus aabbcc is a theorem. Similarly, we can show that $a^n b^n c^n$ ($n > 0$) are theorems in this theory.

In general, the more axioms we use, the more theorems we can prove. However, adding an obvious axiom into the premise does not increase the number of theorems. For example, consider $q(aa,bb,cc) \leftarrow$. We can derive it from the original axioms and thus adding it as a new axiom does not increase the number of theorems. So if we can prove the same set of theorems from two sets S_1 and S_2 of axioms and $S_1 \subseteq S_2$, then we say that S_1 is more essential than S_2.

For a theory T, let $L(T)$ denote the set of the theorems we can prove with T. Note that, given T, $L(T)$ is fixed and there exist infinitely many theorems in $L(T)$. In this sense, once we fix a theory, no new information is added. In this sense, deductive reasoning is said to be truth-preserving.

Using the set of theorems, we can compare different theories. In the case of two theories, there are three types: one theory is included in the other one, they have a subset of common theories but each one is not included in the other one, and they do not have common theories. Given two theories T_1 and T_2, if $L(T_1) \subseteq L(T_2)$, then we say that T_1 is part of T_2.

As shown in the above example of a formal language, when we prove some theorem, the process of the proof is step-by-step and there is no logical gap between each derivation. For example, we obtained formula (6) by axiom (1) and the previous formula (5).

4 Proposed Model of Information Dissemination

In this section, we define a process of information dissemination after we introduce some basic notions. These definitions are premises of our proposed theory. When we send some information, two players, a sender and receiver, exit. Therefore, we consider these players perspectives.

In the following, we assume we already know recognition and do not deeply consider what it is. In other words, we just use recognition as part of building blocks.

4.1 Sender's Perspective

Firstly, a a target information to be sent, we begin to define a vision.

Definition 1 (Vision) *A* vision *of a sender is defined as something recognized by sender's System 1 and has a strong motive to disseminate it.*

There is a wide variety of recognition by System 1. For example, recognition of color or smell is done by System 1. But we do not have motives to tell such recognition to others. So such recognition is not a vision.

An inspiration at an eureka moment is a typical vision. For example, when Steve Jobs first saw a GUI computer at Palo Alt Research Center, he said as follows [10]:

> It was like a veil being lifted from my eyes. I could see what the future of computing was destined to be.

As shown in the definition of vision, we know that such recognition has a strong motive for dissemination. In fact, in the above quote, we see that he was excited and this exciteness can be a motive. Another example of a vision is deep emotion. When we listen to music with deep emotion, we usually have a strong motive to share the emotion with friends.

As described above, inspirations are typical visions, and you may think that only genuine people can obtain inspiration. However, intuition is nothing more and nothing less than recognition in psychology [7]. Therefore, anyone can have inspiration.

We chose the term "vision" for recognition by System 1 because our eye-sight is a typical recognition by System 1, and inspirations are often used with words related to eyesight. We find "see" in the above quote by Jobs. In addition to that, after this famous quote "Creativity is just connecting things", Steve Jobs said as follows [11] (emphasis added):

> When you ask creative people how they did something, they feel a little guilty because they didn't really do it, they just *saw* something.

From this, we can see that he considered an inspiration as a vision.

We should emphasize that a vision is not logically correct because it is recognized by System 1 and there can be some logical leap.

Next, we consider communication of messages, where a *message* represents contents delivered through communication between a sender and receivers. In this sense, a message is a digital data. On the other hand, we assume that something recognized our System 1 can not be expressed with digital data. In this sense, we assume that a vision is analog data.

Some messages sent by a sender are based on some visions whereas other messages are not. To distinguish these two types of messages, we define a message of the former type as a *mission*. The term "mission" originally meant of assignment, indicating the existence of someone who assigns some task to others. To assign tasks, a mission must be expressed clearly to tell the target task without ambiguity. In this sense, the mission must be expressed in the form of digital data. Our definition of a mission is suitable for such situations.

As described in Section 3, to "inform" is to give a form to something recognized by our System 1. Using a newly defined vision and mission, we obtain the following definition of "inform".

Definition 2 *To inform is defined to transform a vision to a mission.*

Once we transform a vision into a mission, we can deliver the vision to others using the transformed mission.

It is well known that when some analog data is transformed into digital one, some information is lost since analog data corresponds to real numbers while digital data does to integers. Therefore it is essentially difficult to inform, that is, to transform a vision into a mission.

4.2 Reciever's Perspective

In the previous section, we have just considered notions about senders. Now we turn to receivers to receive some messages from a sender.

First, we begin with information dissemination. Roughly speaking, it is information spreading. But, in some cases, even if a receiver gets a message from a sender, it does not lead to some attitude or behavioral change of the receiver. In this case, we can think that a message is not recognized receiver's System 1 whereas a vision, which is the target to be sent, is originally recognized by sender's System 1. We consider that information dissemination completes only if messages sent by a sender reach the receiver's System 1.

Definition 3 *Information dissemination is a process to send someone's vision to receivers, where they recognize the vision by their System 1.*

A typical example of this process is as follows: you read a book and are deeply impressed; so you would like to share this feeling with your close friends, by recommending this book to them. If this process is successful, similar emotions will be shared with them. On the other hand, if you do not succeed, your friends do not alter their attitude and behaviors.

The main target of this paper is disruptive changes in a higher education institution. We assume that such a change will change members' attitudes to the target of the change. As a result, the culture and common sense in the institution will be changed, and our goal is to model this process. To this end, we need to define notions related to culture or common sense, which seems difficult to define mathematically. But we first re-define individual System 1 using "being automatic", which can be measured objectively, and then extend the notion to a group of many people.

System 1 and 2 are well-known concepts in psychology, but re-definition enables us to extend these notions from individual recognition to recognition in a group.

Definition 4 (System 1) *If some recognition is done automatically, then we say that the recognition is done by System 1.*

In this definition, "being automatic" is used, and thus now System 1 and 2 are not binary classifications, but they show a continuous spectrum from being completely automatic to requiring full attention. So now we can consider semi-automatic recognition. For example, if you are very good at playing the piano, then you can play some tunes fully automatic, meaning that it is done by System 1 fully. If you are in the middle of training on some tune, you have to pay some attention to playing the tune, meaning that some parts are done by your System 1 but other parts require your attention, System 2.

System 1 and 2 are individual recognition. But, we can extend the idea of System 1 to a group of members because now there are continuous levels of automatic and similarly, there are different numbers of members.

Definition 5 (System 1 for a group) *For a group of members, if recognition by many members in the group done automatically, then we say that this recognition is done by the group's System 1.*

Culture, common sense, organizational values, and identity are typical examples of the group System 1. A major difference between these notions and the group System 1 is that the latter notion is defined as just being automatic and, in principle, it can be measured objectively. It is crucial when we try to evaluate defined notions deductively.

Finally, we describe our daily behaviors as a theory. In our daily life, given a stimulus from outside, we choose some action among some options of actions. In this process, we use both System 1 and System 2 in general. But we are not aware of the recognition of our System 1 because it operates automatically. So we can treat our System 1 and its recognition as premises for our choice. That is, our behaviors and recognition of System 1 constitute a theory, where recognition of System 1 plays a role in premises and a behavior a result of inference, that is a theorem.

5 Proposed Framework and Evaluation

We have to evaluate the proposed model, including notions defined in the previous section. In this section, we evaluate theoretical definitions after explaining how to evaluate them.

First, we introduce fruitfulness as a measure to evaluate. When we construct a theory, we can freely choose or define any concepts for premises. But, the introduced premise can not be proved to be true from this theory. Therefore, we need some other criteria to evaluate premises. In mathematics, being fruitful is often used as such a criterion [12]. We can also find a similar idea in the definition of "charisma" in sociology, where Weber said after some criteria to classify charisma as follows [13]:

> The usefulness of the above classification can only be judged by its results in promoting systematic analysis.

For a theory, a premise in it is said to be *fruitful* if, with this premise, we can prove many, useful theorems in this theory. After Example 1, we considered $q(aa, bb, cc) \leftarrow$ as an another axiom. This is a typical non-fruitful premise because this is derived from the existing axioms. In case of the relativety theory, Einstein introduced "the speed of light is constant" as an important premise, and showed that many surprising predictions were derived from the premises, including it. Therefore, this premise is considered to be valid. Note that many predictions were verified using data by other researchers.

This approach to evaluate some definition is completely different from one of a descriptive theory, which is popular in many fields of social science. To depict this difference, let's consider a leader as an example to be defined because there exist many different definitions about leadership [14].

In our approach, a leader should be defined using facts and premises, and evaluate the definition by theorems derived from the premises. In general, this definition does not cover some people believed to be a leader by someone whereas it may treat some people as a leader, who are not considered as leaders (see Figure 3).

In case of a descriptive theory, we need to describe all the people considered as leaders. But, what is considered a leader varies from person to person, and thus it is difficult to precisely define such a vague target. In fact, Bass and Bass said as follows [15] (emphasis added):

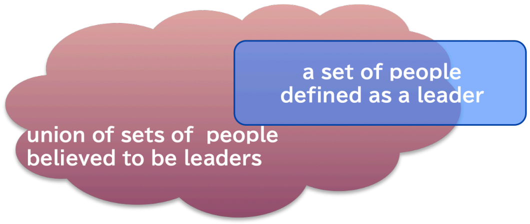

Figure 3: Illustive image of the difference between a descriptive theory and our theory

> the search for the one and only proper and true definition of leadership seems to be *fruitless*.

Even if we focus on a subarea of leadership, the same problem may occur.

Our framework consists of the proposed model of information dissemination and evaluation methods as above. We evaluate the proposed model, assuming the model as premise and deriving some findings from them. If these findings are original results, then we can conclude the definitions are fruitful.

First of evaluation, we derive disruptive changes from the proposed model.

Theorem 6 (Transformation) *If a process of information dissemination for a vision is completed in a group, then it causes a transformational change in the group.*

Before proving the above theorem, we should note that our model does not include the term discuptions or similar words explicitly.

Assume that someone perceives a vision and disseminate it to others in a group. If this process is completed, then many members of the group have affected in their System 1. Therefore, their behaviors are also affected since their premises, that is, their System 1, have changed. So, they have constructed a theory different from an existing one. Using an existing theory, generally speaking, can not evaluate a new theory because their premises are different. Thus we can conclude that this type of changes are transformation.

For comparison, let's consider that a theorem, a behavior, is derived from an existing premise. In this case, we can obtain this theorem through step-by-step proofs, and thus we can not say that the newly derived behavior is disruptive.

In the above theorem, we assumed that dissemination was completed and a new vision was accepted by others. However, human beings have a bias to reject something new. A part of it is known as Semmelweis reflex. Our model can explain the bias as follows:

Theorem 7 *Oppositions against a new vision are true under existing premises.*

The term "bias" implicitly implies that it is based on wrong recognition. However, this theorem says that oppositions against a new vision are always supported by some old premises, and thus such oppositons are inevitable.

Our proposed model of information dissemination can result in minority influence, which is influence of the majority by a minority group [16]. In a process of this type influence, it is known that consistency of a minority group is important. In our model, if a vision has strong motives, then its sender can be consistent to the vision. Therefore, our model explains part of the process of minority influence.

Another important finding derived from our model is a cascade of information dissemination.

Theorem 8 *A process of information dissemination for a vision can cause a new process of information dissemination.*

Assume that a sender A send a message to a receiver B. In our model, such a process starts from recognition in our System 1. Therefore, when B recognizes what is sent by A in System 1, B feels similar emotions or feelings, and it can be a motive to share these feelings to others. Thus, B can be another sender of this message if the process completed. In this way, a cascade happens in our model. This is also supported by computer simulation [17], which shows that cascades happen even if dissemination starts from a minority group. This is different from the theory of minority influence, which basically explains interaction from a minority group to the majority, and does not explain explicitly about cascades.

6 Conclusion

We defined information dissemination and theories, the former one was defined using basic notions of psychology and the latter one was used to express our behaviors. Our main contribution is that we have introduced a mathematical approach to concepts which are usually studied in social science.

Someone might think that the proposed model is too simple to describe real phenomena. In fact, the proposed model is simple, using simple notions such as System 1 and 2. But, any good model is basically simple and is capable of wide application. The proposed model can derive many interesting findings, including transformational changes are derived. These findings from the proposed model are unique and thus the proposed model is proved to be fruitful.

Our definition of transformation starts from a vision, which was defined as recognition of our System 1, such as an inspirations. As described in Section 3, we are able to have inspirations. Contrary to this fact, where you can have an inspiration or not at some situation heavily depends on experience and knowledge you have. Even if you see an apple falling from a tree, it is not always true that you will perceive the law of gravity. From this point, a disruptive change can be creative because, in our proposed model, such a change begin with recognition of someone's System 1, which is unique to you. In this sense, a disruptive change of our proposed model is completely different from restructuring with cost-cutting.

In addition, diversity in an organization is important from the viewpoint of diverse stimulus for inspirations, and many trial-and-errors are essential since a transformation is not based on the existing premises, such as common sense, but creating new premises.

In addition, a simple model can play as a compass for future. You can judge if some trial will cause disruptive changes or not, based on the proposed model. In case of an inductive approach, you can be careful about irregular data since some other premise can explain such data even if an existing premise can not, and such a new premise could lead to transformation. If you can understand it using existing premises, the plan may not create a transformational change because it can derive from the existing premise.

In [5], it is inductively shown that many big farms, which formerly had occupied large part of the market, lost their monopolistic power because they had been relied on existing premises. Our proposed framework can explain such processes from a different perspective.

Acknowledgments

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