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Solution Methods for Unit Commitment Problem Considering Market Transactions

Hiroto Ishimori *, Ryusei Mikami *
Tetsuya Sato *, Takayuki Shiina *

Abstract

In Japan, the electric power market has been fully deregulated since April 2016, and many Independent Power Producers have entered the market. Companies participating in the market conduct transactions between market participants to maximize their profits. When companies consider maximization of their profit, it is necessary to optimize the operation of generators in consideration of market transactions. However, it is not easy to consider trading in the market because it contains many complex and uncertain factors. The number of participating companies continues to increase, and research on the operation of generators in consideration of market transactions is an important field. The power market comprises various markets such as the day-ahead and adjustment markets, and various transactions are performed between participants. We discuss the day-ahead market trading. In such a market, electricity prices and demands vary greatly depending on the trends in electricity sell and purchase bidding. It is necessary for business operators to set operational schedules that take fluctuations in electricity prices and demand into account. We consider an optimization model of generator operation considering market transactions and apply stochastic programming to solve the problem. In addition, we demonstrate that scheduling based on the stochastic programming method is better than conventional deterministic planning.

Keywords: Market transaction, Optimization, Stochastic programming, Unit commitment

1 Introduction

The electric power market has been fully deregulated in Japan, and many independent power producers have entered the market. Companies conduct transactions between market participants to maximize their profits. When companies consider maximization of their profits, it is necessary to optimize the operation of generators in consideration of market transactions.

Cerisola et al.[1] proposes a method for optimizing generator operations for electric utilities in electricity deregulation markets. Also, Shiina and Watanabe[2] propose a method to solve the unit commitment (UC) problem considering the market transactions using the Lagrangian relaxation method. Also, Fukuba et al.[3] proposes a method to solve an opti-

* Waseda University, Tokyo, Japan

mization model for operation planning using stochastic programming by introducing photovoltaic power generation as renewable energy.

However, it is not easy to consider trading in the market because it contains many complex and uncertain factors. The number of participating companies continues to increase, and research on the operation of generators in consideration of market transactions is an important field.

The power market comprises various markets such as the day-ahead and adjustment markets, and various transactions are performed between participants. We assume the day-ahead market trading at Japan Electric Power Exchange (JEPX). In such a market, electricity prices and demands vary greatly depending on the trends in electricity sell and purchase bidding. It is necessary for business operators to set operational schedules that consider fluctuations in electricity prices and demand. Depending on the shortage or excess of power according to the plan, the electricity supplier buys or sells power in the market. In the case of a shortage of power in the market, the price may rise, and the supplier has to pay a substantial amount to buy the power.

In the day-ahead market, the daily market is divided into 30-minutes zones, and the supply side and the demand side bid at each time zone in a situation where the market status of other participants is unknown. A demand and a supply curve are created by accumulating the bidding amount of each company in the price. The price and the electric energy at the intersection of the curves are the transactions actually performed in the market; the price is termed as the contract price, and the transaction volume at that time is the contract volume.

Offer bids with a price lower than the contract price and a purchase bid with a price higher than the contract price are traded at the contract price, and offer bids with a price higher than the contract price and purchase bids with a price lower than the contract price are rejected. The demand curve moves right when the volume of bids rises. The supply curve moves left when the volume decreases. In these cases, the contact price rises.

Further, fuel prices are affected by various factors in the market. We consider the total bidding volume on the demand and supply sides, the fuel cost and the effect of the passage of time, and the fluctuations in the contract price and volume due to the fluctuation of the total bidding volume in a short period of time. Also, the electricity demand are are affected by various factors such as temperature, weather and disaster. Therefore, estimation of the electricity demand is so difficult.

We consider an optimization model of generator operation taking market transactions into account and applying stochastic programming to solve the problem. In addition, we show that scheduling based on the aforementioned method is better than conventional deterministic planning. Mikami et al. [6] proposed a solution to unit commitment problem considering market transaction .In this model, we improve computational efficiency with L-shaped method based on stochastic linear programming problem with recourse and linear approximation.

2 Demand and Supply Curves

The contract price and volume are determined from the demand curve formed by the buying bid on the demand side and the supply curve formed by the selling bid on the supplier side. However, JEPX discloses the total bid volume, contract volume, and contract price in each market every hour, but does not disclose the bidding volume and price of each participant.

Therefore, actual demand and supply curve data cannot be obtained. The demand and supply curves are estimated based on the data released by JEPX[9], and we present the supply and demand curve model of JEPX obtained by the estimation. This study uses the model by Yamaguchi [9].

Random variable

q_b^M Total purchase bidding volume[MWh/h]

Variable

$price_s$ Offer bid price[$ /MWh]

$price_b$ Purchase bid price[$ /MWh]

q_s Offer bid volume[MWh/h]

q_b Purchase bid volume[MWh/h]

Parameter

q_s^M Total offer bidding volume[MWh/h]

f Fuel price[$ /t](use LNG price for fuel price)

t Trend term (natural number that increases by 1 with the passage of business days)

a_0 Constant term in the supply curve

a_1 Coefficient of offer bidding volume in the supply curve

a_2 Coefficient of total offer bidding volume in the supply curve

a_3 Coefficient of fuel cost in the supply curve

b_0 Coefficient of constant term in the demand curve

b_1 Coefficient of bidding purchase volume in the demand curve

b_2 Coefficient of total purchase bidding volume in the demand curve

b_3 Coefficient of trend term in the demand curve

b_4 Coefficient of fuel price in the demand curve

Supply curve

$$price_s = a_0 + a_1 \cdot q_s - a_2 \cdot q_s^M + a_3 \cdot f \tag{1}$$

Demand curve

$$price_b = b_0 + b_1 \cdot b_s - b_2 \cdot q_b^M + b_3 \cdot t + b_4 \cdot f \tag{2}$$

Price and volume

$$price = \frac{b_1(a_2 \cdot q_s^M - a_0 - a_3 \cdot f) - a_1(b_2 \cdot q_b^M - b_0 - b_3 \cdot t - b_4 \cdot f)}{a_1 - b_1} \tag{3}$$

$$volume = \frac{(a_0 - a_2 \cdot q_s^M + a_3 \cdot f) - (b_0 - b_2 \cdot q_b^M + b_3 \cdot t + b_4 \cdot f)}{b_1 - a_1} \tag{4}$$

We derive the contract price and volume based on the estimated demand and supply curves. By identifying the intersection point of the curves, the price at the point where $q_s = q_b$ becomes the contract price, and the volume of transactions at that time becomes the contract volume. Fig.1 shows the relationship between demand and supply curves.

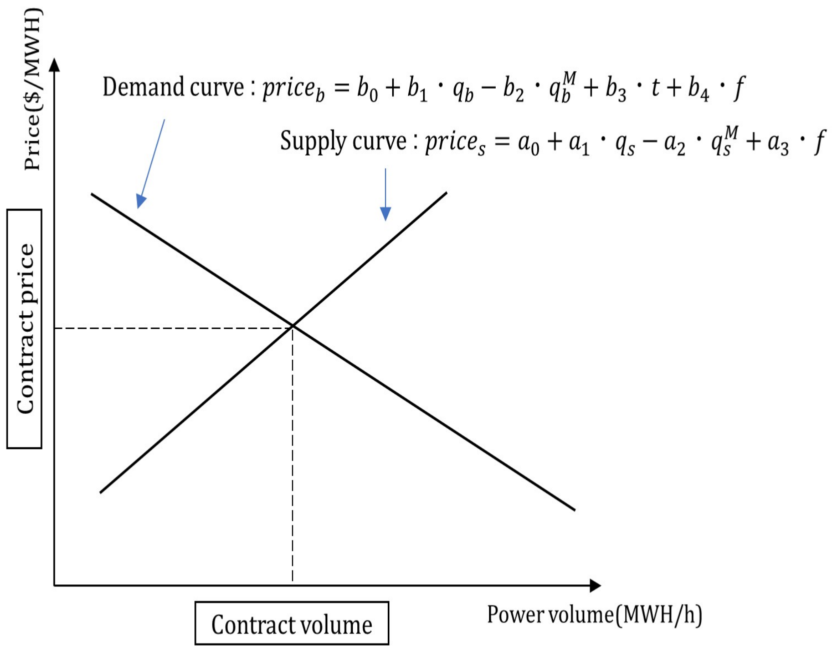

Figure 1: Demand and supply curves

3 Model Formulation

The purpose of the generator operation plan in consideration of market transactions is to satisfy the operation constraints of the generators while taking fluctuations in electricity prices and demand into account, calculating the costs necessary for the operation of the generators, such as fuel and startup costs, from the revenues of the company, and maximizing (profit) excluding electricity purchase costs.

The conventional UC problem without considering the market requires that all the electricity demands be met by the own generator. In such a case, a new generator is often operated additionally to meet the peak demand, which leads to an increase in the total cost. When considering market transactions, the company can purchase electricity from other businesses. Depending on the value of the demand, the total cost can be reduced by supplementing the electricity demand by purchasing electricity from the market rather than by starting a new generator. Therefore, when the total output of the generators operated by the utility is less than the demand, the shortage can be purchased from the market. The generator commitment pattern is the same for all scenarios, and the generator output and electricity changes for scenario s. The formulation and definition of symbols used is described in the following subsections.

3.1 Definition

Variable

x_{it}^s Output of unit i at time period t in scenario s

y_t^s Power purchase amount at time t in scenario s

u_{it} Variable indicating the commitment state of unit i at time t

Parameter

I Number of units

T Time

S Number of scenarios

p_s Probability of scenario s ($\sum_{s=1}^{S} p_s = 1$)

K_t^s Electricity price at time t in scenario s

d_t^s Electricity demand at time t in scenario s

Q_i Maximum output of unit i

q_i Minimum output of unit i

R_i Upper limit of the output fluctuation of unit i

r_i Lower limit of the output fluctuation of unit i

L_i Minimum time that must be continuously on when unit i starts

l_i Minimum time that must be continuously off when unit i stops

Function

$f_i(x_{it}^s)$	Fuel cost in unit i(quadratic function of x_{it}^s)
$g_i(u_{i,t-1}, u_{i,t})$	Startup cost for unit i(function of u_{it})

3.2 Formulation

$$(UC) \quad \min \sum_{s=1}^{S} p_s \left(\sum_{t=1}^{T} K_t^s y_t^s + \sum_{i=1}^{I} \sum_{t=1}^{T} f_i(x_{it}^s) u_{it} \right) + \sum_{i=1}^{I} \sum_{t=1}^{T} g_i(u_{i,t-1}, u_{i,t}) \tag{5}$$

s.t.

$$\sum_{i=1}^{I} x_{it}^s + y_t^s \geq d_t^s, t = 1, \cdots, T, s = 1, \cdots, S \tag{6}$$

$$y_t^s \geq 0, t = 1, \cdots, T, s = 1, \cdots, S \tag{7}$$

$$q_i u_{it} \leq x_{it}^s \leq Q_i u_{it}, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{8}$$

$$r_i \leq x_{it}^s - x_{it-1}^s \leq R_i, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{9}$$

$$u_{it} - u_{i,t-1} \leq u_{i\tau}, \tau = t+1, \cdots, \min\{t + L_i - 1, T\}, i = 1, \cdots, I, t = 2, \cdots, T \tag{10}$$

$$u_{i,t-1} - u_{it} \leq 1 - u_{i\tau}, \tau = t+1, \cdots, \min\{t + l_i - 1, T\}, i = 1, \cdots, I, t = 2, \cdots, T \tag{11}$$

$$u_{it} \in \{0, 1\}, i = 1, \cdots, I, t = 1, \cdots, T \tag{12}$$

The objective of (5) is the minimization of the expected value of the cost. Cost is the sum of the electricity purchase, fuel, and startup costs. Since the commitment state does not change for each scenario, the startup cost does not consider the occurrence probability of each scenario. On the contrary, the fuel and electricity purchase costs consider the occurrence probability of each scenario since these values change for each scenario.

Inequality (6) is a constraint that the sum of the total output of the units and the electricity purchase amount at time t in scenario s satisfies the electricity demand.

Inequality (7) is a constraint that the electricity purchase amount at time t in scenario s is greater than or equal to 0.

Inequality (8) are the upper and lower limits of generator output, while inequality (9) indicates those of the output fluctuation of the generator.

Inequality (10) indicates the minimum time that must be continuously on after unit i is turned on, and inequality (11) indicates the minimum time that must be continuously off after unit i is turned off.

Constraint (12) represents the 0–1 condition of the decision variable u_{it}. If the total power output by unit i is less than the demand at time t in scenario s, it is assumed that the demand is compensated by purchasing electric power.

4 Solution

4.1 L-shaped method

We used the L-shaped method, which is used to efficiently obtain a solution to a stochastic programming problem including second decision variables, to solve the UC problem. Muckstadt and Koenig [7] used a deterministic model for the UC problem, while Takriti et al. [8] showed a model that considered the variation in electricity demand.

We first form the basic two-stage stochastic linear programming problem with recourse as (SPR).

$$
\begin{array}{rl}
\text{(SPR): min} & c^\top x + \mathscr{Q}(x) \\
\text{subject to} & Ax = b \\
& x \geq 0 \\
\text{where} & \mathscr{Q}(x) = E_{\tilde{\xi}}[Q(x,\tilde{\xi})] \\
& Q(x,\xi) = \min\{q^\top y(\xi) \mid Wy(\xi) \geq \xi - Tx, y(\xi) \geq 0\}, \xi \in \Xi
\end{array}
$$

In the formulation of (SPR), c is a known n_1-vector, b a known m_1-vector, $q(>0)$ a known n_2-vector, and A and W are known matrices of size $m_1 \times n_1$ and $m_2 \times n_2$, respectively. The first stage decisions are represented by the n_1-vector x. We assume the m_2-random vector $\tilde{\xi}$ is defined on a known probability space. Let Ξ be the support of $\tilde{\xi}$, i.e. the smallest closed set such that $P(\Xi) = 1$.

Given a first stage decision x, the realization of random vector ξ of $\tilde{\xi}$ is observed. The second stage data ξ become known. Then, the second stage decision $y(\xi)$ must be taken so as to satisfy the constraints $Wy(\xi) \geq \xi - Tx$ and $y(\xi) \geq 0$. The second stage decision $y(\xi)$ is assumed to cause a penalty of q. The objective function contains a deterministic term $c^\top x$ and the expectation of the second stage objective. The symbol $E_{\tilde{\xi}}$ represents the mathematical expectation with respect to $\tilde{\xi}$, and the function $Q(x,\xi)$ is called the recourse function in state ξ. The value of the recourse function is given by solving a second stage linear programming problem.

It is assumed that the random vector $\tilde{\xi}$ has a discrete distribution with finite support $\Xi = \{\xi^1, \ldots, \xi^S\}$ with $\text{Prob}(\tilde{\xi} = \xi^s) = p^s, s = 1, \ldots, S$. A particular realization ξ of the random vector $\tilde{\xi}$ is called a scenario. Given the finite discrete distribution, the problem (SPR) is restated as (DEP), the deterministic equivalent problem for (SPR).

$$
\begin{array}{rl}
\text{(DEP): min} & c^\top x + \sum_{s=1}^{S} p^s Q(x,\xi^s) \\
\text{subject to} & Ax = b \\
& x \geq 0 \\
\text{where} & Q(x,\xi^s) = \min\{q^\top y(\xi^s) \mid Wy(\xi^s) \geq \xi^s - Tx, y(\xi^s) \geq 0\}, s = 1, \ldots, S
\end{array}
$$

To solve (DEP), an L-shaped method (Van Slyke and Wets) has been used. This approach is based on Benders decomposition. The expected recourse function is piecewise linear and convex, but it is not given explicitly in advance. In the algorithm of the L-shaped method, we solve the following problem (MASTER). The new variable θ denotes the upper bound for the expected recourse function such that $\theta \geq \sum_{s=1}^{S} p^s Q(x,\xi^s)$.

$$
\begin{array}{rl}
\text{(MASTER): min} & c^\top x + \theta \\
\text{subject to} & Ax = b \\
& x \geq 0 \\
& \theta \geq 0
\end{array}
$$

Let x^*, θ^* be the optimal solution of (MASTER), then the following second stage problem is solved for $s = 1, \ldots, S$.

$$
\begin{aligned}
Q(x^*,\xi^s) &= \min\{q^\top y(\xi^s) \mid Wy(\xi^s) \geq \xi^s - Tx^*, y(\xi^s) \geq 0\} && (13) \\
&= \max\{(\xi^s - Tx^*)^\top \pi(\xi^s) \mid \pi(\xi^s)^\top W \leq q^\top, \pi(\xi^s) \geq 0\} && (14)
\end{aligned}
$$

If minimization problem (13) is infeasible for some scenario ξ^s, the optimal objective value of maximization problem (14) is infinite above or problem (14) is infeasible. Leaving out the latter case, we have a dual solution $\bar{\pi}(\xi^s) \geq 0$ which satisfies the following inequalities.

$$(\xi^s - Tx^*)^\top \bar{\pi}(\xi^s) > 0 \quad \text{and} \quad \bar{\pi}(\xi^s)^\top W \leq 0 \tag{15}$$

To cut off solution x^*, the feasibility cut (16) is added to the formulation of (MASTER).

$$(\xi^s - Tx)^\top \bar{\pi}(\xi^s) \leq 0 \tag{16}$$

If minimization problem (13) is feasible for $\forall \xi \in \Xi$ and $\theta^* < \sum_{s=1}^{S} p^s Q(x^*, \xi^s)$, let $\pi^*(\xi^s)$ be the solution of problem (14). In this case, the optimality cut (17) is added as an outer approximation of $\sum_{s=1}^{S} p^s Q(x, \xi^s)$.

$$\theta \geq \sum_{s=1}^{S} p^s (\xi^s - Tx)^\top \pi^*(\xi^s) \tag{17}$$

As can be seen from Fig. 2, by repeating this, the optimality cuts are added to the master problem. The feasible region of the first decision variable is limited in order to calculate the exact value of the objective function, which is a piecewise linear convex function. The optimal solution of the stochastic programming problem including the second decision variable can be obtained by repeating these iterations.

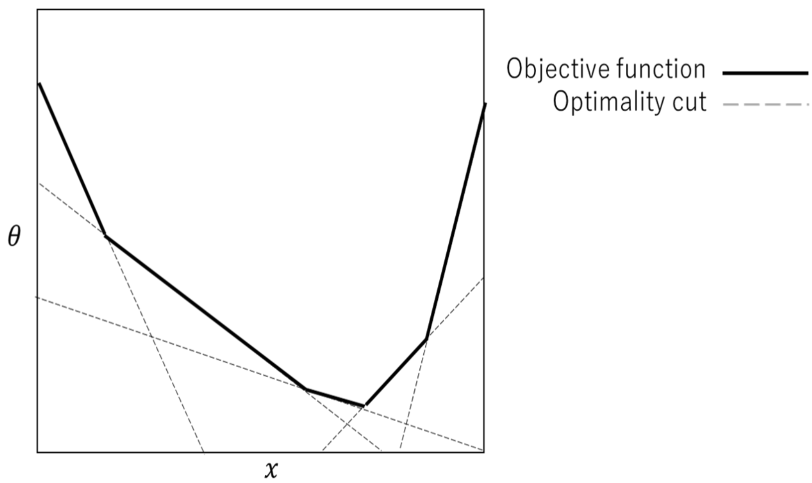

Figure 2: Optimality cut

Moreover, a linear approximation can be applied to the formulation to reduce the solution time. Since the function of fuel cost is a convex quadratic function, the calculation time can be reduced by converting it from a quadratic programming problem to a mixed-integer programming problem using linear approximation.

4.2 Formulation

Parameters to add

$NCUT$ Total number of optimality cuts

φ Total fuel and electricity purchase costs

φ_s Sum of fuel and electricity purchase costs in scenario s

θ_{it}^s Linear approximation of fuel cost of unit i at time period t in scenario s

$\alpha_{it}^{s,ncut}$ Coefficient of u_{it} in the optimality cut

$\beta_{it}^{s,ncut}$ Constant term in the optimality cut

a_i^1, a_i^2 Coefficient of x_{it}^s in the linear approximation

b_i^1, b_i^2 Constant term in the linear approximation

Master problem (First stage)

$$\min \sum_{i=1}^{I} \sum_{t=1}^{T} f_i(0) u_{it} + \sum_{i=1}^{I} \sum_{t=1}^{T} g_i(u_{i,t-1}, u_{i,t}) + \varphi \tag{18}$$

s.t.

$$u_{it} - u_{i,t-1} \leq u_{i\tau}, \tau = t+1, \cdots, \min\{t + L_i - 1, T\}, i = 1, \cdots, I, t = 2, \cdots, T \tag{19}$$

$$u_{i,t-1} - u_{it} \leq 1 - u_{i\tau}, \tau = t+1, \cdots, \min\{t + l_i - 1, T\}, i = 1, \cdots, I, t = 2, \cdots, T \tag{20}$$

$$u_{it} \in \{0, 1\}, i = 1, \cdots, I, t = 1, \cdots, T \tag{21}$$

$$\varphi \geq \sum_{s=1}^{S} p_s \theta_s, s = 1, \cdots, S \tag{22}$$

$$\varphi_s \geq \sum_{i=1}^{I} \sum_{t=1}^{T} \alpha_{it}^{s,ncut} u_{it} + \beta_s^{ncut}, ncut = 1, \cdots, NCUT, s = 1, \cdots, S \tag{23}$$

$$\varphi, \varphi_s \geq 0, s = 1, \cdots, S \tag{24}$$

Second stage

$$C_s(u) = \min \sum_{t=1}^{T} K_t^s y_t^s + \sum_{i=1}^{I} \sum_{t=1}^{T} \theta_{it}^s \tag{25}$$

s.t.

$$\sum_{i=1}^{I} x_{it}^s + y_t^s \geq d_t^s, t = 1, \cdots, T, s = 1, \cdots, S \tag{26}$$

$$y_t^s \geq 0, t = 1, \cdots, T, s = 1, \cdots, S \tag{27}$$

$$q_i u_{it} \leq x_{it}^s \leq Q_i u_{it}, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{28}$$

$$r_i \leq x_{it}^s - x_{it-1}^s \leq R_i, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{29}$$

$$\theta_{it}^s \geq a_i^1 x_{it}^s + b_i^1, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{30}$$

$$\theta_{it}^s \geq a_i^2 x_{it}^s + b_i^2, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{31}$$

$$\theta_{it}^s \geq 0, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{32}$$

Costs that do not vary with the scenario and those that vary with the scenario are divided into the master problem and second problem, respectively.

(22) is the overall optimality cut, and (23) is the optimality cut in scenario s. (24) is a constraint that the sum of the fuel and electricity purchase costs in scenario s is greater than or equal to 0. (30) indicates the linear approximation of unit i at time t in scenario s at the minimum output, and (31) indicates the linear approximation of unit i at time t in scenario s at the maximum output. (32) is a constraint that the linear approximation of the function of the fuel cost of unit i at time t in scenario s is greater than or equal to 0.

Also, the algorithm of L-shaped method in this study is shown in Table 1.

Table 1: Algorithm of L-shaped method

Step1	Set the provisional value of objective function $\bar{z} := \infty$, the lower bound value $\underline{z} := 0$
Step2	Solving master problem obtains optimal solution $\hat{u}_{it}$ and $\hat{\varphi}$
Step3	If $\sum_{i=1}^{I}\sum_{t=1}^{T} f_i(0)\hat{u}_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} g_i(\hat{u}_{i,t-1}, \hat{u}_{i,t}) + \hat{\varphi} > \underline{z}$, set $\underline{z} := \sum_{i=1}^{I}\sum_{t=1}^{T} f_i(0)\hat{u}_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} g_i(\hat{u}_{i,t-1}, \hat{u}_{i,t}) + \hat{\varphi}$. If $\sum_{i=1}^{I}\sum_{t=1}^{T} f_i(0)\hat{u}_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} g_i(\hat{u}_{i,t-1}, \hat{u}_{i,t}) + \sum_{s=1}^{S} C_s(\hat{u}) < \bar{z}$, set $\bar{z} := \sum_{i=1}^{I}\sum_{t=1}^{T} f_i(0)\hat{u}_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} g_i(\hat{u}_{i,t-1}, \hat{u}_{i,t}) + \sum_{s=1}^{S} C_s(\hat{u})$.
Step4	If $\bar{z} \leq (1+\varepsilon)\underline{z}$, the algorithm terminates.
Step5	If $\hat{\theta}_{it}^{s} < C_s(\hat{u})$, $\forall s \in S$, add optimality cut to master problem and return to Step2.

4.3 Optimality Cut

After obtaining the first decision variable $\hat{u}_{it}$ by solving the master problem, the second stage problem for each scenario is solved to obtain the second decision variable $\hat{x}_{it}^{s}, \hat{y}_{t}^{s}, \hat{\theta}_{it}^{s}$. The optimality cut is calculated using the optimal dual solution $\hat{\pi}_t^s$, $\hat{\lambda}_{it}^s$, $\hat{\mu}_{it}^s$, $\hat{\rho}_{it}^{s,1}$, $\hat{\rho}_{it}^{s,2}$ of (26), (28), (30), (31). The dual problem is as follows.

$$\max_{\lambda_{it}^s, \mu_{it}^s, \pi_t^s, \rho_{it}^{s,1}, \rho_{it}^{s,2}} \min_{x_{it}^s, y_t^s, \theta_{it}^s} \sum_{i=1}^{I}\sum_{t=1}^{T} (\theta_{it}^s + f_i(0)) u_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} g_i(u_{i,t-1}, u_{i,t}) +$$
$$\sum_{i=1}^{I}\sum_{t=1}^{T} (b_i^1 \rho_{it}^{s,1} + b_i^2 \rho_{it}^{s,2}) + \sum_{t=1}^{T} \pi_t^s d_t^s + \sum_{i=1}^{I}\sum_{t=1}^{T} (\mu_{it}^s q_i - \lambda_{it}^s Q_i) u_{it} \tag{33}$$

s.t.

$$\lambda_{it}^s, \mu_{it}^s, \pi_t^s, \rho_{it}^{s,1}, \rho_{it}^{s,2} \geq 0, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{34}$$
$$x_{it}^s, y_t^s, \theta_{it}^s \geq 0, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{35}$$
$$r_i \leq x_{it}^s - x_{it-1}^s \leq R_i, i = 1, \cdots, I, t = 1, \cdots, T, s = 1, \cdots, S \tag{36}$$

Using these dual solutions, the following inequality becomes a valid inequality for the master problem.

$$\varphi_s \geq \sum_{i=1}^{I}\sum_{t=1}^{T} (b_i^1 \rho_{it}^{s,1} + b_i^2 \rho_{it}^{s,2}) + \sum_{t=1}^{T} \hat{\pi}_t^s d_t^s + \sum_{i=1}^{I}\sum_{t=1}^{T} (\hat{\mu}_{it}^s q_i - \hat{\lambda}_{it}^s Q_i) u_{it} \tag{37}$$

The coefficients α,β of (23) can be defined as follows.

$$\alpha_{it}^{s,ncut} = \hat{\mu}_{it}^{s} q_i - \hat{\lambda}_{it}^{s} Q_i \tag{38}$$

$$\beta_{s}^{ncut} = \sum_{i=1}^{I}\sum_{t=1}^{T}(b_i^1 \rho_{it}^{s,1} + b_i^2 \rho_{it}^{s,2}) + \sum_{t=1}^{T} \hat{\pi}_t^s d_t^s \tag{39}$$

4.4 Linear Approximation

Mikami et al.[6] calculated the fuel cost as a quadratic function; in this study, the fuel cost is approximated linearly to reduce the calculation time. The function of the fuel cost is expressed as $f_i(x_{it}^s) = A_i {x_{it}^s}^2 + B_i x_{it}^s + C_i$, so θ_{it}^s can be calculated by linear approximation at the lower and upper limits of the output. The largest difference between the linear approximation and the quadratic function is 0.3%; therefore, approximation does not make a significant difference to the problem. Fig. 3 shows the linear approximation of the fuel cost function. By linear approximation, the fuel cost function is approximated from quadratic function to linear function.

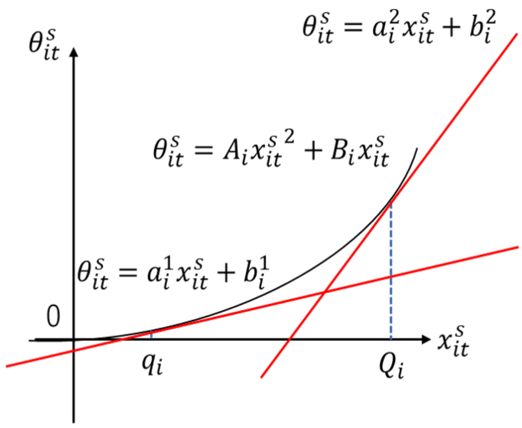

Figure 3: Linear approximation

5 Computional Experiments

The contract price and the fixed volume are affected by the total bid volume. We assume a four days operation plan for the experiment. Due to the short period, fluctuations in fuel prices are not considered in this study. The scenario of electricity price and demand is represented by a scenario tree that branches into two every day. The experimental period was four days; the number of scenarios therefore becomes eight, and the occurrence probability of each scenario s is (1/8).

When considering market transactions, the electricity price and demand fluctuate according to the tender trends of market participants. Since the daily JEPX market is divided into 30 minutes zone, the number of time zones per day is 48 with 30 minutes as one unit. Similarly, numbers are assigned in the order of time zone 1 from 0:00 to 0:30 and time zone

2 from 0:30 to 1:00. For the experiment, we refer to the transaction data for December 2005 published by JEPX. Since the generator operation plan period is four days, the number of time zones is 192.

The electricity price and demand are created with reference to the average total bid amount and that for each time period of the day in December in 2005. A scenario of contract price and contract quantitative change is generated using demand and supply curves to consider the fluctuation of electricity price and demand in the market. Knowledge of the contract price and contracted volume allows us to predict the demand for the individual business operator. We assume that there will be a certain ratio of the demand for the personal business operator to the contracted amount. Fig. 4 shows the scenario tree in this model. This scenario tree is divided into two per day.

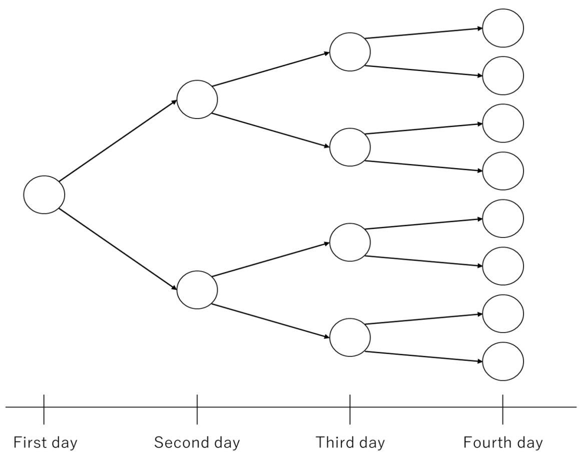

Figure 4: Scenario tree

6 Result

6.1 Effectiveness of Stochastic Programming

We compare a model based on stochastic programming with a deterministic model. Birge and Louveaux [5] defined the evaluation of stochastic programming solutions. For the value of the solution of the stochastic programming problem, VSS, the value of the stochastic solution is used. For the cost minimization problem, VSS is defined as VSS = EEV − RP. Recourse problem (RP) is the optimal objective function value of the stochastic programming problem. The expected result of using the expected value problem (EVV) solution is an optimal objective function value when a plan is made based on the average value of random variables.

In the case of a UC problem, the EEV uses the unit's state of each time zone of the generator obtained when solving the problem with the average value of the random variable. Using the solution of the problem with average data, the EEV can be obtained. The starting and stopping states of the generator at each time zone obtained when solving the problem based on the average value of the random variables are not optimal for stochastic problem. Therefore, from the relationship between RP and EEV, RP $\leq$ EEV, VSS $\geq$ 0 holds. Assuming that the electricity demand that the operator must meet in each time zone will be 50% to

150% of the contract volume, the generator operation plans for each demand is compared. The LNG price used for the fuel price, which is a parameter included in the demand and supply curves during the operation planning period, is assumed to be 400 [$/t]. In addition, we assume the company has eight generators. Table 2 shows the relationship between RP and VSS. As the ratio increases, VSS becomes larger. We solved the problem using CPLEX 12.10.0.0.

Table 2: Cost of RP and EEV

Ratio of demand to a contract volume	RP($)	EEV($)	VSS($)
50%	59,160	60,892	1,732
75%	86,941	91,038	4,097
100%	11,385	115,712	4,327
125%	131,071	138,153	7,082
150%	157,584	166,921	9,337

A model using stochastic programming that considers electricity demand and price fluctuations has higher profits than a deterministic model and is suitable for operation in an actual market where electricity demand and price fluctuations occur. Furthermore, since the schedule of units is fixed regardless of scenario s, the amount of electricity purchased increased as the ratio of demand to a contract volume increased.

6.2 Efficiency of the Solution

We compared the problem solved using the L-shaped method and linear approximation with the direct method. It can be seen that solving the problem in this way leads to a significant reduction in calculation time. Further, the objective function value can be calculated with a certain degree of accuracy.

Table 3: L-shaped solution

Number	Number of	Solution Time	
of scenarios	time zones	Direct (s)	L-shaped (s)
1	48	891	200
2	96	6990	41
4	144	10665	277
8	192	64176	498

Table 3 indicates that for all the problems, the L-shaped method, which is more efficient in terms of calculation time, has a shorter calculation time than the direct method. Comparing the computation time of the direct and L-shaped methods, the computation speed is approximately 4.5 times faster when the number of scenarios is 1. However, when the number of scenarios is 8, the computation speed is approximately 130 times faster. This indicates that as the number of scenarios per time period increases, the effect of reducing the calculation time for the direct method becomes greater. While we set up a scenario with two branches in one day.

However, if we set up a scenario with more branches in a shorter period of time and increase the number of scenarios in relation to the number of time periods, the computation time of the proposed model will be more efficient.

7 Conclusion

We proposed a UC model that considers the market transactions required after electricity liberalization. Cost was lower than the generator operation planning model that did not consider market transactions, indicating that it was an excellent model for operators aiming to minimize cost. 8 scenarios are not enough to consider realistic uncertainty so we need to consider larger model than this model. The L-shaped method and linear approximation lead to a significant reduction in calculation time. In the future, we would like to extend on a solution method that can handle large-scale problems with an increased number of scenarios and facilities.

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