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Dynamic Subscription Pricing for Digital Service Platform

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Abstract

This research explores the dynamics of subscription-based revenue models, with a particular emphasis on the long-term pricing of the freemium model in digital service markets. By employing a continuous-time dynamic control model, the study examines how companies can strategically determine subscription prices to maximize overall profitability. The analysis focuses on the relationship between subscription fees, user base segmentation, and company investments in product enhancements. The findings indicate that subscription pricing are essential for converting free users to paid subscribers and lowering operational costs. Moreover, the results offer valuable insights for digital service providers on optimizing their pricing decisions to boost platform profitability.

Keywords: Dynamics, Subscription pricing, Digital services

1 Introduction

In recent years, the subscription-based business model has become widely adopted across various digital platforms, offering services ranging from streaming media to software as a service (SaaS). Among these, the freemium approach has attracted significant interest due to its distinctive method of user acquisition and revenue generation. This model provides a basic service without charge while reserving advanced features for paying customers. This strategy not only enables rapid user base growth but also presents considerable challenges in converting free users to paying customers and optimizing revenue streams. The advent of digital technology has transformed how companies deliver services, with the subscription model becoming increasingly common across various industries. This model's rise is particularly notable in digital platforms, where services range from media streaming to cloud-based solutions. Among the numerous revenue models, the freemium strategy stands out for its distinctive approach to user engagement and monetization. This model offers basic services at no cost while reserving premium features for those who pay.

The success of the freemium model largely hinges on its ability to balance user acquisition with the conversion of free users to paying customers, a challenge that has garnered

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substantial academic and industry attention. This research delves into the intricacies of subscription strategies, focusing specifically on the long-term impacts of different subscription tiers on service improvements and user conversion rates. Utilizing a dynamic control model, we explore various scenarios, including traditional subscriptions and innovative models supported by advertisements, to derive optimal strategies aimed at maximizing profitability and enhancing user satisfaction.

2 Literature Review

The digital era has significantly transformed consumer behavior and business models, leading to the increasing popularity of the subscription-based approach, particularly within the freemium model framework. This study is related to the two research streams: dynamic pricing and freemium models.

Dynamic Pricing

Dynamic pricing has significantly evolved with technology, moving beyond traditional price negotiations. Businesses now use algorithms and data analytics to optimize revenue and offer tailored shopping experiences by adjusting prices dynamically [4]. Sui et al. [8] examined that platforms have shifted from subsidies and low-price strategies to investing in value-added services (VAS) to expand user bases. This study examines two competing platforms with bilateral users who can multi-home. Using a two-stage game model, they analyze dynamic pricing decisions when VAS is offered. In the first stage, platforms may subsidize or charge users, favoring subsidies for sides with weaker indirect network effects (INEs). In the second stage, platforms should charge all users, offering lower VAS levels and higher access fees to one side while providing higher VAS levels and lower fees to the other side. Liu et al. [5] addressed the dynamic pricing, replenishment, and rationing (DPRR) challenges faced by omni-channel retailers under uncertain demand. They develop a model to maximize profits, proposing a Maskable Long-Short-Term-Memory Proximal Policy Optimization (ML-PPO) algorithm that integrates observed and predicted states. Experiments show that the ML-PPO algorithm effectively improves profits and service levels. This research fills a gap in understanding joint pricing, ordering, and inventory rationing in omni-channel retail, providing valuable insights for managing uncertain demand environments. Sheng et al. [7] discussed that optimal pricing strategies for retailers facing informed (strategic) and uninformed consumers. Key findings are that dynamic pricing suits markets with fewer uninformed consumers, while pre-announced pricing fits moderate patience levels. In decentralized supply chains, dynamic wholesale pricing benefits manufacturers, retailers, and consumers by mitigating double marginalization. Dynamic pricing is crucial for platforms and retailers to optimize profits and manage user bases effectively. It allows businesses to adapt to varying demand and consumer behavior. Studies show that platforms benefit from dynamic pricing by initially subsidizing users and later charging all users with differentiated value-added services. For omni-channel retailers, dynamic pricing, combined with strategic inventory management, helps balance demand and inventory, reducing mismatches and maximizing profits. Moreover, in markets with informed and uninformed consumers, dynamic pricing is more effective for those with fewer uninformed consumers, while pre-announced pricing suits markets with moderate patience levels. However, previous research has not utilized dynamic evolutionary models to investigate dynamic pricing issues, highlighting the academic value of this study.

Freemium Subscription

The freemium model, which offers core services for free while charging for premium features, has become a staple in the digital economy, driven by its potential to enhance user base and profitability [2, 6]. Thus, freemium is crucial for digital businesses as it attracts a large user base by offering basic services for free while monetizing through premium features. This model enables rapid growth and user acquisition, providing valuable data and engagement metrics. Freemium also allows businesses to upsell premium features, creating a scalable and sustainable revenue stream. Its flexibility and appeal to a wide audience make freemium an effective strategy in competitive digital markets. Tyrväinen and Karjaluoto [9] indicated that in freemium model, users can familiarize themselves with the service, reducing uncertainty and building trust in the provider. This increased trust can lead to the purchase of premium features. Freemium also supports monetization through virtual items, such as in-game assets or additional functionalities. Thus, they focused on understanding the impact of perceived value on users' willingness to pay (WTP) for premium features and how trust mediates this relationship. and also examined different monetization strategies within the freemium model, such as feature limitations and virtual item sales, to identify effective ways to maximize revenue and user conversion rates. Hamari et al. [3] empirically investigated the impact of perceived service quality on consumer behavior in freemium services, particularly free-to-play games. It addresses a critical gap in existing research by focusing on how perceived service quality influences both continued use and premium purchases. Their study employs data from an online survey of 869 participants collected through gaming-related websites and social media pages for a deeper understanding of the dynamics between service quality perceptions and consumer behavior in the freemium business.

The literature underscores the importance of dynamic pricing and freemium in the success of digital services. Dynamic pricing allows businesses to adapt to market demands and consumer behavior, optimizing revenue. It is vital for digital service providers to refine their freemium offerings and achieve sustainable market growth. This study adopts a dynamic control model to discuss that how a platform remains competitive, attractive, and capable of meeting evolving user needs while maximizing profitability.

3 The Model

This study investigates a scenario where a platform considers introducing a freemium policy to quickly attract a base of interested users. We examine two cases: Case B, where only subscribers can access the platform service, and Case F, where the freemium model is adopted, allowing free users to access basic services and subscribers to access premium services. The platform adapts its subscription pricing dynamically based on current market conditions. At any time t, the platform decides on the subscription price p. This pricing decision influences the subscription demand d, and the cumulative subscription amount S evolves over time, starting from zero. The cumulative subscription amount S can enhance users' utility to subscribe due to network effects. If the freemium model is applied, users who pay the subscription fee gain access to exclusive features, while others use the free version of the service. The platform's expenses at any time include variable costs associated with the number of free users. The platform's revenues consist of earnings from advertisements viewed by free users r and income from users subscribing to the premium service. The study develops a continuous-time dynamic control model over the time interval $t \in [0, \infty]$, requiring the platform to continuously tailor its strategies to evolving market

conditions.

When the freemium model is adopted, the platform offers both basic (free) and premium subscription services, generating income through advertising and premium subscriptions from two distinct user groups. Users' decisions to subscribe are influenced by the price and the cumulative subscription amount. The utility function for users subscribing at time t is defined by:

$$U^S(t) = \phi - \alpha p(t) + \varepsilon S(t) \tag{1}$$

where ϕ indicates the user valuation following a uniform distribution $g(\phi) \sim U[0,1]$, α is the sensitivity of users to the subscription price, and ε represents the intensity of network effects related to the cumulative subscription amount. Thus, subscription amount and free-user amount at time t are $d_s(t) = \int_{\{\phi: U^S(t) \geq 0\}} g(\phi) d\phi$ and $d_f(t) = \int_{\{\phi: U^S(t) < 0\}} g(\phi) d\phi$, respectively.

The decision variable is the subscription price $p(t)$. The state variable is represented by the cumulative subscription amount, $S(t)$. Moreover, the instantaneous evolution of $S(t)$ follows the dynamics below:

$$\dot{S}(t) = \frac{dS(t)}{dt} = d_s(t) - \sigma S(t); \quad S(0) = 0 \tag{2}$$

The instantaneous rate of the subscription amount includes the current subscription demand and undergoes a certain degree of reduction, denoted by σ, where $0 < \sigma < 1$, signifies the decay rate of the system's subscription amount. As the cumulative subscription amount augments, the rate of this subscription amount decay correspondingly increases. This phenomenon indicates that a portion of the subscribers will leave the system. To counteract this, the platform has to adjust the subscription price. The cost associated with each free user is articulated $f(d_f(t))^2$, establishing a framework to evaluate the financial implications of serving free activities. This cost function indicates that when the amount of free users is greater, the cost will dramatically increase, and thus it is harmful to the platform, leading the platform has to balance the income from advertising and free-user cost.

The objective for the platform is to maximize total profit. This objective encapsulates the strategic balance between incurring costs for free service and generating income through premium services. Under Case B, the free services are unavailable, and the instantaneous profit of the platform is $\pi^B = p(t)d_s(t)$; under Case S, the freemium policy is adopted, and thus the platform will be $\pi^S = p(t)d_s(t) + r d_f(t) - f(d_f(t))^2$. The revenue from users opting for the premium service is given by $p(t)d_s(t)$, while $r, d_f(t)$ represents the advertising revenue from free users. The platform's pricing decisions affect the proportion of users choosing between free and premium services, necessitating a strategic approach to optimize overall revenue. As the cumulative subscription amount $S(t)$ grows, users' willingness to subscribe increases. The variable cost for free users is represented by $f(d_f(t))^2$, indicating that a large number of free users will significantly increase the platform's burden. Therefore, it is crucial for the platform to carefully analyze the dynamics between cost and revenue to ensure profitability. Regarding the framework of the objective function, the parameter ρ denotes the intertemporal discount rate. This rate diminishes the value assigned to future expected rewards, suggesting that a higher ρ directs the manufacturer towards a more immediate gains perspective. Without this rate, the calculation of future benefits could potentially result in unbounded values, making the discount rate essential in models that span an infinite horizon. The formulation of the dynamic control model is expressed

mathematically as Equation (3).

$$\max_{p(t)} \Pi^j = \int_0^\infty e^{-\rho t} \pi^j \, dt; \quad s.t \begin{cases} \dot{S}(t) = d_s(t) - \sigma S(t); \\ p(t) \geq 0; \\ S(0) = 0, \quad j \in \{S, B\}. \end{cases} \tag{3}$$

Employing the Bellman equation, the platform aims to maximize the immediate profit available under the state $S(t)$, considering the current decisions $p(t)$ and anticipating the future rewards that these decisions yield. By iteratively refining this process, the platform can ascertain the optimal strategies, $\hat{p}(t)$. The Bellman equation is expressed as Equation (4):

$$\rho V^j(S(t)) = \max_{p(t)} \{\pi^j + \left(\frac{d}{dS(t)} V(S(t))\right) (d_s(t) - \sigma S(t))\}. \tag{4}$$

Solving the first-order derivative, the instantaneous equilibrium price of Case j can be obtained, as follows:

$$\hat{p}^S = \frac{1 - \alpha V' + 2\alpha f S(t)\varepsilon + \alpha r + S(t)\varepsilon}{2\alpha + 2\alpha^2 f}; \quad \hat{p}^B = \frac{1 - \alpha V' + S(t)\varepsilon}{2\alpha}. \tag{5}$$

Considering Case S for illustration, we substitute (5) into (4) and obtain

$$\begin{aligned} \rho V^S(S(t)) = & \frac{\alpha^2 r^2 - 2\alpha r(S(t)\varepsilon - 1) - \alpha^2 (\lambda_2 + 2\lambda_1 S(t))^2 + (S(t)\varepsilon + 1)^2}{4\alpha\rho} \\ & + \frac{1}{2} (\lambda_2 + 2\lambda_1 S(t)) (\alpha\lambda_2 - \alpha r + 2\alpha\lambda_1 S(t) - 2\sigma S(t) + S(t)\varepsilon + 1) \end{aligned} . \tag{6}$$

Solving the value function V of each case equal to zero, the final equilibrium decision can be obtained.

4 Analysis

Next, we conduct numerical simulations based on the following fundamental parameters considered in previous literature: $\alpha = 0.6$, $\varepsilon = 0.15$, $r = 0.1$, $f = 0.06$, $\rho = 0.3$, $\sigma = 0.25$. Figure 1 shows the trends of the cumulative subscription amount and the price with respect to time. Figure 1(a) shows that the cumulative subscription amount $S(t)$ increases over time. Interestingly, the freemium model does not necessarily guarantee an increase in the number of subscribers. In fact, nearing a steady state, not offering freemium (Case B) can result in a higher amount of subscribers. This is why many platforms are reluctant to offer a freemium strategy. However, this does not mean that the freemium strategy is ineffective. The following analysis will focus on profitability for further illustration.

Next, we consider the change of the platform decision in time. In Figure 1(b), we observe that initially, the pricing without a freemium strategy is higher than with a freemium strategy. This is mainly because, without free users, there is no advertising revenue, and the platform must increase subscription prices to maintain revenue. However, the freemium model allows the platform to initially lower subscription fees to expand the subscriber base.

(a) State: cumulative subscription amount, $S(t)$ (b) Decision: Subscription price $p(t)$

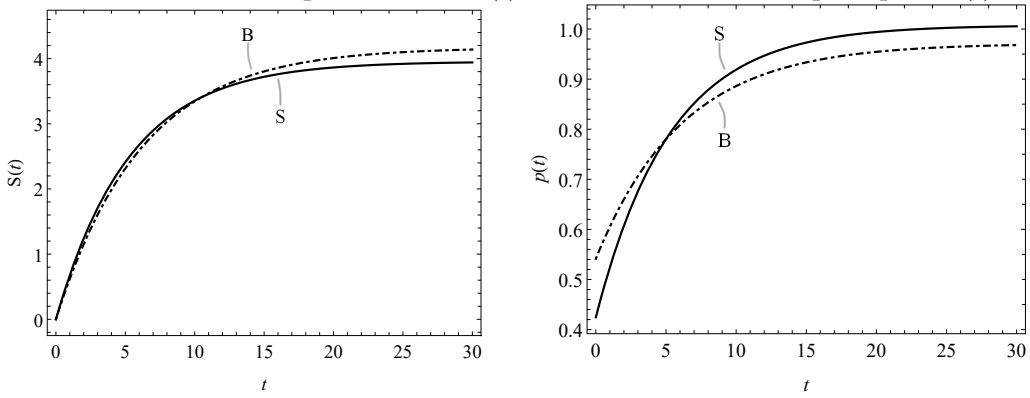

Figure 1: Trend of the state and decision with respect to time

As the amount of subscribers increases, the network effect grows. At this point, the subscription price under the freemium model becomes higher because there is already a substantial subscriber base. The platform can gradually increase subscription revenue while reducing advertising revenue. This result indicates that the freemium model provides the platform with more flexible pricing options, allowing for more significant dynamic price adjustments based on market conditions.

Figure 2(a) depicts the changes in profitability over time. We can observe that the freemium model is not always beneficial; the profits at both the initial and long time are lower compared to not having a freemium model. This indicates that the benefits of freemium become apparent only when the platform reaches a certain subscription amount. However, as time progresses, the benefits gradually diminish because the costs associated with free users hinder profitability. Therefore, in the long run, focusing on premium services might be more advantageous. We will plot a two-dimensional contour plot to illustrate the platform's preferences by comparing the cumulative profits in Figure 2(b). It demonstrates that when the decay rate of subscribers (σ) is high, not adopting a freemium model is a better strategy because the free plan can erode the subscriber base, and the platform cannot effectively retain subscribers. Conversely, when the platform can effectively maintain the amount of subscribers and reduce the decay rate, offering a freemium model becomes an effective strategy that can increase long-term cumulative profits. However, we observe an extreme scenario when the decay rate is very low, and the platform is short-sighted (not valuing long-term profits, i.e., ρ is high). In this case, not offering a freemium model becomes the preferred strategy. This is because the freemium model leverages network effects to bring future benefits, but the future benefits generated by the freemium model are not desired by the platform, which places more importance on the immediate revenue from subscribers.

5 Summary

This study focuses on the 'freemium' business model, where platforms offer basic services for free and charge for advanced features. This model is popular in the software, multimedia services, and gaming industries. It aims to quickly build a large user base through free services and then monetize through premium subscriptions or advertising. Our research

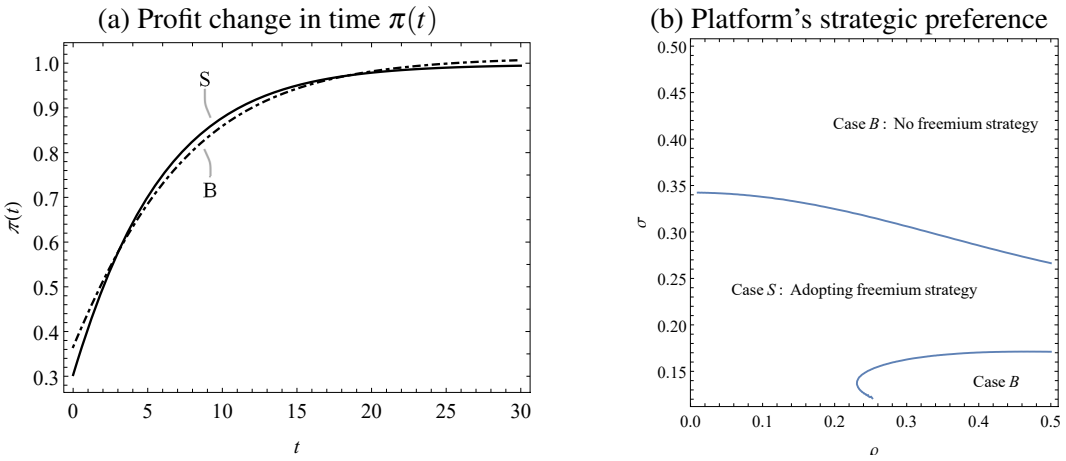

Figure 2: Profit change in time and the platform's preference

targets companies operating under this model, investigating how they can adapt to varying market conditions and modify their strategies over time to optimize revenue from subscriptions and advertisements. In this analysis, we explore how a platform dynamically adjust price in market conditions and investigate the platform's long-term profit. We compare dynamic outcomes across these scenarios to better understand whether to provide freemium strategy.

The model reveals that while the freemium strategy can initially lower subscription fees to attract a broader user base, it also introduces complexities in balancing costs and revenues from advertisements and premium subscriptions. Over time, the cumulative subscription amount increases, and the network effect intensifies, allowing the platform to adjust prices dynamically. However, the analysis indicates that the freemium model does not always guarantee higher subscriber numbers or profits. In fact, in the long term, focusing on premium services might be more beneficial as the costs associated with free users can hinder profitability. Numerical simulations show that without freemium, subscription prices are higher initially due to the absence of advertising revenue. As the subscriber base grows, the freemium model allows for more flexible pricing adjustments, reflecting the dynamic market conditions. Profitability analysis reveals that freemium benefits become apparent only after reaching a certain subscriber threshold. Moreover, platforms with high subscriber decay rates may find freemium less advantageous due to the increased cost burden from free users. Overall, the study highlights the strategic considerations digital platforms must account for when implementing subscription models. While freemium can enhance user acquisition and provide flexible pricing strategies, its long-term effectiveness depends on the platform's ability to maintain and convert its user base efficiently. Balancing immediate revenue needs with long-term profitability is crucial, especially in dynamic and competitive digital markets.

The model in this study is simplified for clarity, leaving room for future improvements to better reflect real-world contexts. Future research could expand on these findings by examining market size changes due to pricing and subscription strategies, exploring the effects of reference prices in competitive markets, and distinguishing between process and product innovations. Additionally, integrating the cost dynamics of standard and different subscription models to identify conditions that favor one over the other, and incorporating a long-term dynamic model in freemium frameworks to better understand user engagement

and monetization strategies, would provide deeper insights.

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References

[1] Chenavaz, R., 2012. Dynamic pricing, product and process innovation. European Journal of Operational Research 222 (3), 553–557.

[2] Gu, X., Kannan, P., Ma, L., 2018. Selling the premium in freemium. Journal of Marketing 82 (6), 10–27.

[3] Hamari, J., Hanner, N., Koivisto, J., Feb. 2017. Service quality explains why people use freemium services but not if they go premium: An empirical study in free-to-play games. International Journal of Information Management 37 (1), 1449–1459.

[4] Kopalle, P. K., Pauwels, K., Akella, L. Y., Gangwar, M., Dec. 2023. Dynamic pricing: Definition, implications for managers, and future research directions. Journal of Retailing 99 (4), 580–593.

[5] Liu, S., Wang, J., Wang, R., Zhang, Y., Song, Y., Xing, L., Jun. 2024. Data-driven dynamic pricing and inventory management of an omni-channel retailer in an uncertain demand environment. Expert Systems with Applications 244, 122948.

[6] Rietveld, J., 2018. Creating and capturing value from freemium business models: A demand-side perspective. Strategic Entrepreneurship Journal 12 (2), 171–193.

[7] Sheng, J., Du, S., Nie, T., Zhu, Y., Nov. 2023. Dynamic pricing vs. pre-announced pricing in supply chain with consumer heterogeneity. Electronic Commerce Research and Applications 62, 101311.

[8] Sui, R., Liu, M., Liu, Y., Zha, X., Jun. 2024. Two-sided dynamic pricing and value-added service investment strategies of competitive platforms considering indirect network effects. International Journal of Production Economics 272, 109262.

[9] Tyrväinen, O., Karjaluoto, H., Aug. 2024. Willingness to pay for freemium services: Addressing the differences between monetization strategies. International Journal of Information Management 77, 102787.