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Development of the Descriptive-Relational-Graphical (DRG) Model for Eduinformatics: Fostering Relational Understanding through Representational Transformation in Data-Driven Mathematics Education

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Abstract

The Organization for Economic Co-operation and Development (OECD) Learning Compass 2030 envisions education as cultivating transformative competencies for navigating uncertain futures. This study proposes the Descriptive-Relational-Graphical (DRG) Model as a comprehensive framework for understanding mathematical learning through representational transformation. The DRG Model integrates three complementary modes: descriptive language for articulation through natural expression, relational language for quantitative and logical structures, and graphical language for spatial visualization. This framework emerged from data-driven education research within Eduinformatics, an interdisciplinary field integrating educational sciences with informatics methodologies. Building on Ainsworth's DeFT framework, Duval's semiotic representation theory, and Skemp's distinction between instrumental and relational understanding, this study demonstrates that mathematical learning develops through recursive movement across representational systems. Visualization functions as a central mediating pathway connecting internal cognition with external representation. Analysis of geometric examples illustrates how bidirectional transformations among descriptive, relational, and graphical representations foster relational understanding—comprehending both what to do and why. The DRG Model supports OECD Learning Compass 2030 goals by enhancing learner autonomy, metacognitive reflection, and agency in mathematics education and beyond.

Keywords: Descriptive-Relational-Graphical (DRG) Model, Representational transformation, Visualization, Relational understanding, Mathematical learning, Eduinformatics, Data-driven education, OECD Learning Compass 2030

1 Introduction

1.1 The OECD Learning Compass 2030

The Organization for Economic Co-operation and Development (OECD) Learning Compass 2030, developed within the Future of Education and Skills 2030 project, envisions education as a process through which learners cultivate the agency and transformative competencies necessary to navigate an uncertain world. It defines learning not merely as knowledge acquisition but as the development of the capacity to act ethically and creatively toward individual and collective well-being. The metaphor of a compass symbolizes the learner's ability to orient themselves autonomously within dynamic and unpredictable contexts, rather than following predetermined directions [1] [2].

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1.2 Reading Literacy and Mathematical Language

In the OECD framework, reading literacy extends beyond decoding text to include interpreting, evaluating, and reflecting upon meaning in diverse contexts [1]. A similar expansion applies to mathematics: mathematical literacy involves understanding and expressing relationships through descriptive, relational, and graphical representations [1]. It connects linguistic reasoning with quantitative and spatial thinking, requiring learners to shift flexibly among these three forms of expression [1]. This multi-representational fluency forms the foundation of mathematical communication and understanding [1].

1.3 Purpose and Theoretical Framework

This study aims to demonstrate that deep academic understanding requires transformation among languages, and that the three representational languages of mathematics—descriptive, relational, and graphical—can serve as an effective model for fostering mutual understanding among linguistic forms.

Building on Ainsworth's Design, Functions, and Tasks (DeFT) framework for learning with multiple representations [3] and Duval's theory of semiotic representation [4], learning is viewed as a process of coordination and conversion between representational registers. Ainsworth emphasizes the complementary functions of multiple representations in supporting learning [3] , whereas Duval highlights the cognitive operations required for transforming meaning across semiotic systems [4]. Each transformation—whether internal reorganization within a mode or external translation among modes—constitutes a step in meaning construction. From this perspective, mathematical comprehension arises not from the accumulation of procedures but from the dynamic transformation that integrates descriptive, relational, and graphical representations into coherent understanding.

1.4 Eduinformatics: A Data-Driven Perspective on Representation and Learning

Eduinformatics provides a data-driven lens for analyzing how learners construct meaning across representational systems. By observing transitions among descriptive, relational, and graphical expressions, Eduinformatics identifies patterns that reveal how cognition and expression interact. This perspective contributes to evidence-based curriculum design that enhances representational fluency and coherence between internal understanding and external articulation [5].

2 Research Questions

The theoretical frameworks discussed above—Ainsworth's DeFT framework, Duval's semiotic representation theory, and Skemp's distinction between instrumental and relational understanding—suggest that deep mathematical learning involves dynamic transformation among multiple representational systems. However, these theories have not been sufficiently integrated to explain how learners actively coordinate and transform representations in ways that align with contemporary educational goals such as those articulated in the OECD Learning Compass 2030.

The primary research question that arises from this context is: **How do transformations among descriptive, relational, and graphical representations foster relational understanding and support the development of learner agency as envisioned by the OECD Learning Compass 2030?**

To address this overarching question, this study pursues three subsidiary inquiries. First, what are the distinct cognitive functions of descriptive, relational, and graphical representations in mathematical learning, and how do these functions complement one another? Second, in what ways does visualization serve as a mediating pathway that enables learners to construct, translate, and integrate meaning across representational systems? Third, how can the bidirectional and recursive nature of representational transformation be leveraged pedagogically to move learners from instrumental understanding to relational understanding?

By answering these questions through theoretical analysis and examination of geometric examples, this study aims to demonstrate that representational transformation constitutes the core mechanism through which mathematical understanding develops.

3 Reconstructing Knowledge

3.1 Knowledge Must Not Become Hollow

When knowledge is reduced to mere accumulation or repetition, learning loses its transformative potential. The OECD notion of transformative competencies—creating new value, reconciling tensions and dilemmas, and taking responsibility—offers a framework to reconstruct knowledge as a living process of reasoning, representation, and expression. Here, knowledge is not static content but a dynamic interaction among symbols, diagrams, and language, through which learners give meaning to what they know and transform it into new understanding.

For deep understanding, spiral learning is essential; as learners revisit core ideas with progressively varied representations, memory becomes a process of reconstruction that naturally yields a 'map of knowledge.' Language plays a major role in this process, for it can be translated into both visual and symbolic forms. This is most evident in the learning of mathematics.

3.2 Relational and Instrumental Understanding Reconsidered

According to Skemp in 1976, instrumental understanding refers to knowing the rules and procedures that yield correct answers, whereas relational understanding involves knowing what to do and why [5] . While instrumental understanding often produces immediate success in problem solving, it easily leads to what may be called hollow knowledge—a fragile competence detached from meaning and structure. In contrast, relational understanding cultivates a coherent network of concepts, enabling learners to integrate new ideas and reconstruct prior knowledge within a unified cognitive schema.

In classroom contexts, this distinction also reveals why students often find mathematics difficult to understand. Two factors are especially significant. First, teachers tend to reproduce their own learning experiences: they teach as they were taught, thereby transmitting not only knowledge but also the same limitations of understanding. Second, the language of mathematics itself can obscure communication. Its symbols and expressions, designed for precision and abstraction, may conceal rather than reveal meaning when not connected to verbal or visual representations.

Therefore, relational understanding is not merely a learner's goal but also a teacher's responsibility—to mediate between mathematical language and learners' intuitive reasoning, and to reconstruct one's own understanding through dialogue. Only by this dual process of reflection and reconstruction can mathematical communication become genuinely meaningful.

3.3 Spiral Learning and the Reconstruction of Understanding

Bruner proposed that "to learn structure is to learn how things are related" in 1960. In this view, genuine understanding arises when learners perceive not isolated facts but the relationships that organize them into coherent systems. A curriculum built upon this principle should revisit fundamental ideas repeatedly, each time at a higher level of abstraction—a process that Bruner termed spiral learning [6].

In such a design, prior knowledge is not merely reviewed but reconstructed. Each return to a familiar concept enables learners to connect symbolic, visual, and verbal representations in new ways, thereby transforming memory from storage into structure [7]. This recursive process aligns with Skemp's notion of relational understanding: learning becomes meaningful when actions are integrated with reasons and when representations are coordinated across levels of abstraction [6] [7].

From an educational standpoint, the spiral principle implies that teaching must not aim at closure but at continuity[6]. Each encounter with a concept opens pathways to reinterpretation, inviting learners to reorganize their prior understanding. Through this dynamic revisiting, mathematical knowledge grows as a living system—a network of relationships that can expand and adapt rather than a static accumulation of procedures [7].

This principle of revisiting and restructuring is clearly observable in the learning of the geometric concept of an angle. Initially, learners recognize an angle visually as a form—an opening or "turn" between two lines. Through measurement with degrees, this perceptual relation becomes quantified. With the introduction of the radian, the idea is abstracted further, linking the arc length s and the radius r through $s = r\theta$, thereby identifying the angle with a real number. Finally, the trigonometric functions on the unit circle integrate this quantitative understanding into an analytic framework, connecting geometry with algebra and calculus.

Therefore, the development of the concept of angle exemplifies Bruner's spiral curriculum: each stage revisits the same idea at a deeper level of abstraction—from perception to measurement, from quantity to structure, and from geometry to analysis [6].

4 Superiority of Visualization

Visualization constitutes a powerful mode of cognition that complements linguistic and symbolic reasoning. In mathematical learning, the ability to externalize abstract relations in visual form enables learners to grasp structure and interconnection that are often concealed in purely verbal or numerical expressions. Bruner emphasized that perception and imagery are fundamental to concept formation [6] in 1960 , while Duval identified visualization as a distinct semiotic register through which mathematical objects become accessible to thought [4] in 1999.

Across domains of human knowledge, visual representation consistently demonstrates superior efficiency in communicating complex information. Contour maps, for example, translate numerical data of elevation into immediate perceptual patterns: the spacing of contour lines conveys the gradient of terrain more intuitively than tables of figures could. Meteorological charts employ isobars and front lines to depict the dynamic behavior of atmospheric pressure systems. Musical notation converts temporal and acoustic events into a two-dimensional diagram that simultaneously expresses pitch, duration, and harmony. Likewise, in investigative or diagnostic contexts, diagrammatic flowcharts clarify causal sequences and relationships that would otherwise require lengthy verbal explanation.

These examples reveal that visualization functions not merely as illustration but as a cognitive transformation—a shift from sequential description to spatial configuration. When learners translate verbal or symbolic data into diagrams, they reorganize information according to structural relations rather than temporal order. This process fosters insight, comparison, and error detection. Visualization thus acts as a mediating form between description and formal symbolism, facilitating the internalization of concepts through perceptual reasoning.

Pedagogically, the superiority of visualization suggests that mathematics instruction should actively engage learners in constructing and interpreting visual representations. Tasks that invite students to see relations—to map, graph, or sketch patterns—encourage integration across the three representational languages: descriptive, symbolic, and diagrammatic. Through such cross-modal activity, learners develop flexible understanding and the ability to navigate among multiple perspectives, a competence that underlies both creative problem solving and relational understanding.

4.1 Conceptual Background

The educational significance of visualization can be traced to a long intellectual tradition that views perception as the foundation of reasoning. Bruner proposed three modes of representation—enactive, iconic, and symbolic—arguing that conceptual understanding develops through the progressive coordination of these modes in 1960. The iconic mode, in particular, bridges concrete manipulation and abstract symbolism by allowing learners to organize experience through images and spatial relations [6]. In mathematics education, this mode corresponds to diagrams, graphs, and other visual structures that mediate between intuition and formal notation.

Duval further advanced this view by distinguishing between representation registers and the transformations among them in 1999. According to his framework, mathematical comprehension does not reside in any single register but emerges from the ability to convert one form of representation into another [4] —for example, from a verbal description to an algebraic expression, or from a formula to a geometric figure. Visualization thus plays a dual role: it provides perceptual access to abstract concepts and enables cognitive flexibility through representational transformation.

Skemp contributed an additional psychological dimension by differentiating between instrumental and relational understanding in 1976. Visualization supports the latter, as it reveals the underlying structures that connect separate procedures. When students draw or imagine diagrams to interpret symbolic rules, they reconstruct meaning relationally rather than mechanically. The act of seeing relations becomes the act of understanding itself [7] .

These theoretical perspectives converge on a common insight: visualization is not an optional aid but a core component of thought. It transforms mathematical reasoning from the manipulation of symbols into the organization of relationships within a visual field. Recognizing this, mathematics education must treat visualization as a mode of reasoning equal in status to linguistic and symbolic representation.

The development of the DRG model presented in this study emerged from the authors' sustained engagement with visualization methodologies in educational contexts. Through collaborative teaching of the first-year required course "Manaburu Tokiwabito" at Kobe Tokiwa University [8], which incorporates logical thinking, critical thinking, and visual thinking into its curriculum, the authors gained pedagogical insights into the cognitive function of visualization in supporting logical reasoning. This practical experience, combined with theoretical investigation into representational transformation, led to the conceptualization of the triangular relationship among descriptive, relational, and graphical modes.

Furthermore, the authors have systematically developed novel visualization methods within the Eduinformatics framework [9], [10]—an interdisciplinary field integrating educational sciences with informatics methodologies. These prior studies include visualization of curricula using syllabus data combined with cosine similarity and multidimensional scaling methods [11], competency-based curriculum mapping [12], [13], and the application of t-SNE (t-Distributed Stochastic Neighbor Embedding) for higher-dimensional educational data visualization [14] [15] [16] [17]. Through this research trajectory focused on making educational structures visible and interpretable, the importance of graphical representation as a mediating cognitive tool became increasingly evident, ultimately leading to the formulation of the DRG model as a comprehensive framework for understanding mathematical learning through representational transformation.

4.2 Superiority of Visualization

Visualized information enables the intuitive and spatial understanding of relationships and patterns of change that are not immediately grasped through numerical or verbal descriptions alone.

Contour lines on a map connect points of equal elevation, and the spacing between the lines indicates the degree of slope: wide intervals represent gentle gradients, while narrow intervals indicate steep ones. By connecting two points on the map with a line segment, it is also possible to create a cross-sectional diagram and to quantify the gradient between them as a ratio. Thus, a contour map functions not merely as a depiction of terrain but as a visual model for interpreting variations and tendencies in elevation (Figure 1).

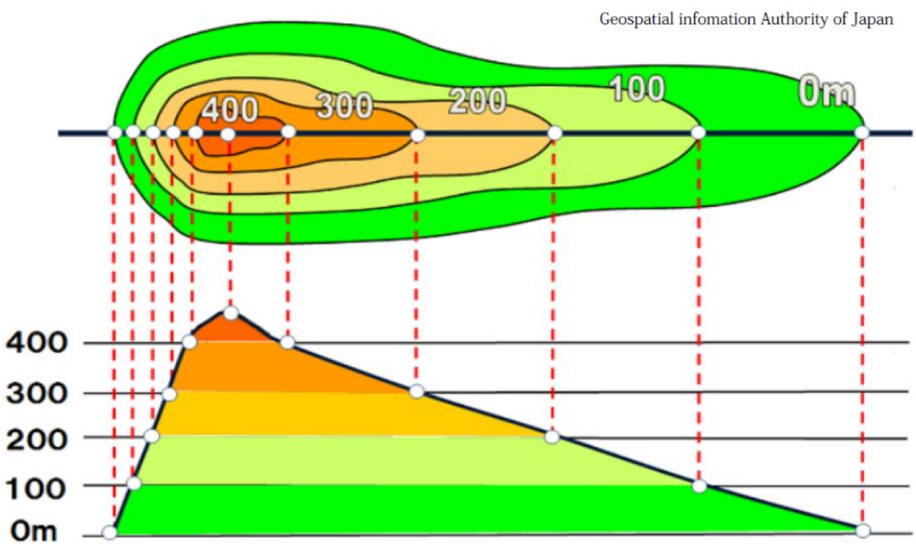

Figure 1: Contour lines on a map

The spacing of isobars on a weather map represents the degree of change in air pressure. Narrowly spaced isobars indicate strong winds, while widely spaced ones suggest weak winds; this allows one to predict the regions and directions of strong airflow. Since winds blow from high-pressure to low-pressure areas, the arrangement of isobars makes it possible to intuitively grasp atmospheric motion. Here again, numerical data such as air pressure are expressed as lines, and through visualization, temporal change and spatial distribution become meaningful patterns.

Musical notation, too, is a form of visualization. The placement of notes on the staff represents pitch, their shape indicates duration, and their combinations reveal harmony and melodic flow. Simply by following the arrangement of notes, one can intuitively perceive rhythm, motion, and even musical expression. A score, therefore, is a visual representation that transforms the temporal sequence of sounds into a spatial configuration, from which performers can read the structure of sound and emotion.

Across these examples—maps, weather charts, and musical scores—certain common features emerge. First, by expressing quantitative relationships (differences in elevation, pressure, or pitch) as spatial forms, one can grasp numerical distributions and patterns of change intuitively. Second, from the configuration of lines and symbols, it becomes possible to interpret relationships, directions of change, and even predict future states. Third, visualization is not a mere visual translation but an intellectual operation that mediates between numerical and verbal information through the language of form.

In sum, visualization represents a transformation from quantity to form meaning, through which the structure and dynamics of information can be discerned. It thus enables the interpretation of relationships, changes, and predictions. Visualization, therefore, should not be regarded as a mere technique of representation, but rather as a cognitive bridge that allows learners to concretize and internalize abstract concepts.

4.3 Geometry and the Formation of Concepts

Geometry provides a vivid example of how linguistic and visual representations diverge. Concepts such as "point," "angle," and "parallel" are usually introduced visually rather than verbally, so that the terminology itself often becomes more difficult than the explanation. Teachers may assert that when lines are parallel, corresponding angles are equal, or conversely, if corresponding or alternate angles are equal, the lines are parallel. Here, angle is treated as a numerical quantity, although its conceptual basis differs from that of length. Whereas length is measurable through proportional relationships based on unit definition, the angle requires a conceptual shift to radian measure, in which the proportionality between arc length and radius defines its magnitude. Understanding this transformation—from visual perception to quantitative abstraction—marks a crucial stage in mathematical cognition.

4.4 Transformation among Representations in Geometry

The process of reasoning in geometry can be described through the Descriptive–Relational–Graphical (DRG) model (Figure 2). In Figure 2, the transformation Descriptive–Graphical (DG) ⇒ Descriptive–Relational (GR) represents the process of expressing a geometric configuration—specifically, a quadrilateral whose diagonals bisect each other—through descriptive, relational, and graphical forms. Students first draw the quadrilateral ABCD and identify the intersection O of the diagonals, engaging in a graphical representation that allows them to recognize symmetry and congruence visually. Describing this configuration as "a quadrilateral whose diagonals bisect each other at their midpoints" transforms the visual schema into a descriptive representation, emphasizing the relationship among parts.

When students reason that the triangles formed by the diagonals have equal sides and equal included angles, they conclude that the opposite sides of the quadrilateral are parallel. This reasoning corresponds to a relational transformation, in which logical connections among lengths and angles are expressed verbally rather than through direct computation (Figure 3).

In Figure 3, the transformation DR ⇒ Relational–Graphical (RG) shows the complementary route—from descriptive reasoning to relational formulation and finally to graphical confirmation.

Students first note that point O is the midpoint of both diagonals and that point P represents the center of the position vectors. From this reasoning, they infer that the opposite sides are equal and parallel, leading to the conclusion that the quadrilateral ABCD is a parallelogram.

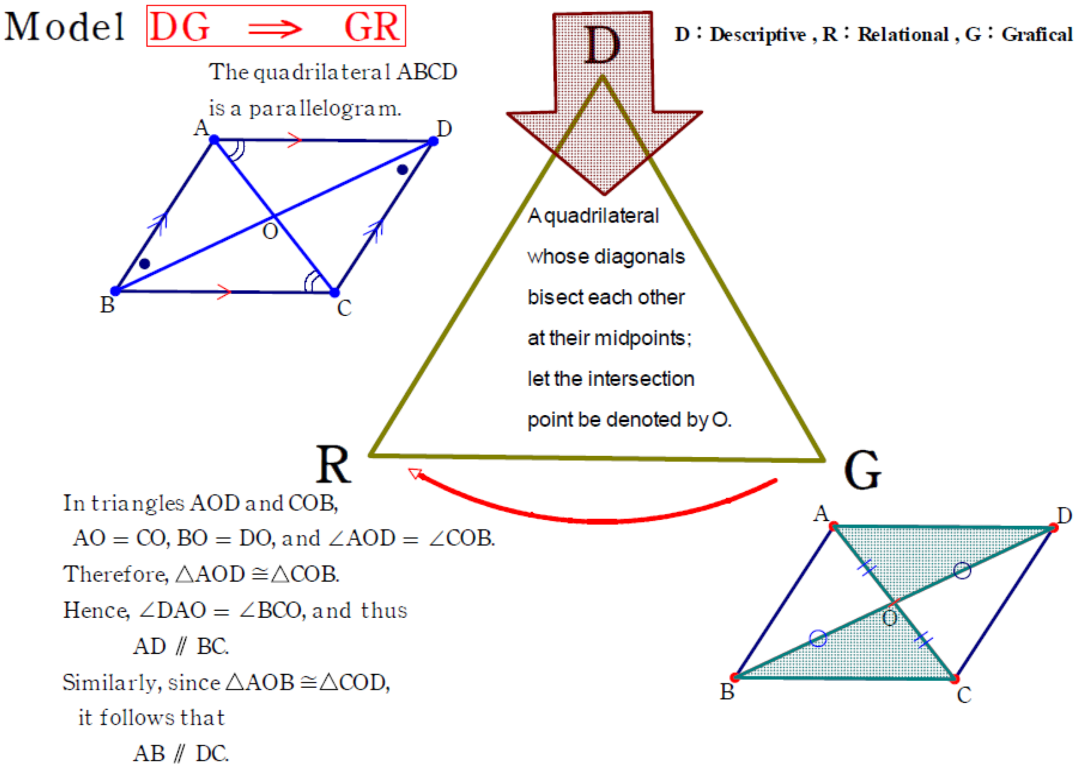

Figure 2: The transformation DG ⇒ GR

Note: Equality of vectors implies equality of both magnitude and direction, whereas the concept of parallelism does not include direction. Together, these figures illustrate how understanding in geometry develops through bidirectional transformations among descriptive, relational, and graphical representations—embodying the transition from instrumental to relational understanding as characterized by Skemp [7].

4.5 Reversibility and Creative Reconstruction

Conversely, this relationship among representations is bidirectional. Starting from an abstract structure, one can reconstruct its visual form. For instance, by drawing two concentric circles centered at O and taking their diameters AC and BD, a parallelogram ABCD can be generated, and the parallelism AD ∥ BC emerges visually (Figure 4). Such reversibility between visualization and formalization indicates not only mastery of procedures but also the ability to reconstruct knowledge creatively—an essential aspect of mathematical literacy and competency. From the perspective of the OECD Learning Compass 2030, these representational shifts foster agency [1]—the learner's ability to navigate and construct meaning autonomously. Thus, the proof of a simple geometric property reveals the deeper cognitive architecture of learning—how visualization, language, and symbolism interact to form understanding

4.6 Pedagogical Implications

The pedagogical implications of visualization lie in its ability to mediate between perception and abstraction. When students are encouraged to visualize mathematical relations, they are not merely drawing pictures but constructing a representational bridge that connects intuitive experience with formal reasoning. Such activity enables learners to recognize underlying structures and to coordinate the three representational languages—verbal, symbolic, and diagrammatic—through active transformation.

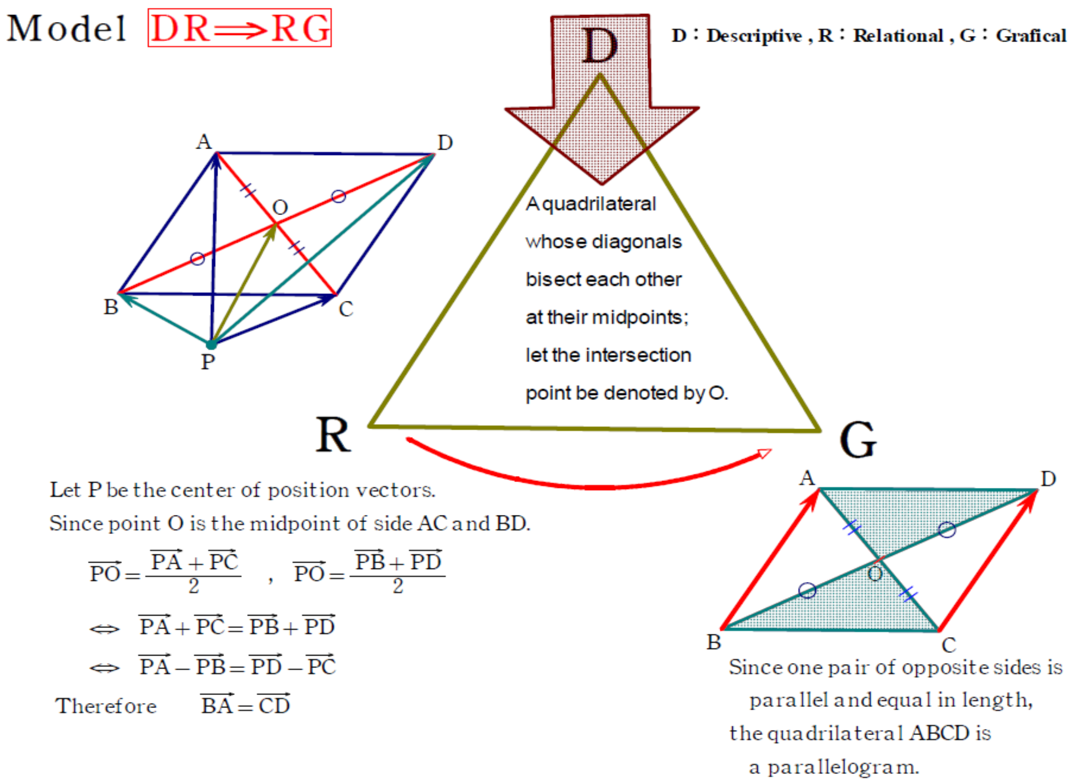

Figure 3: The transformation DR ⇒ RG

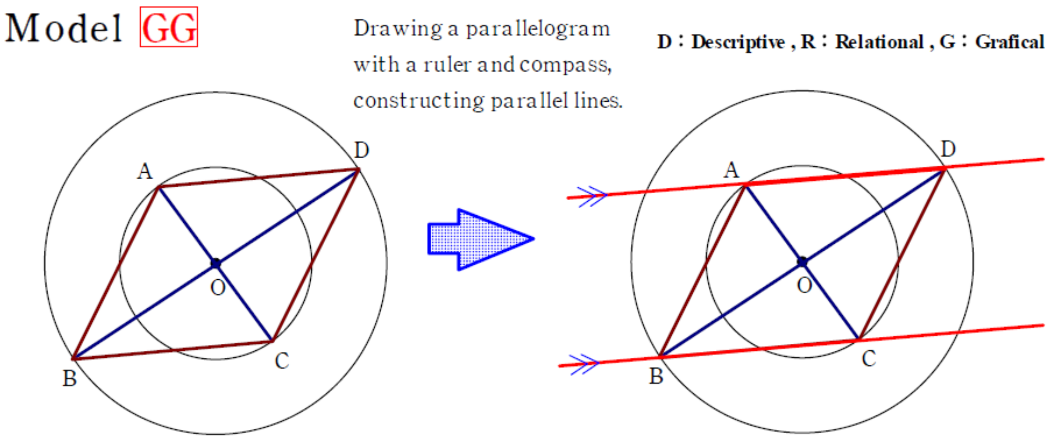

Figure 4: the parallelism AD ∥ BC emerges visually

In the classroom, visualization should be treated as an exploratory tool rather than an illustrative afterthought. Tasks that invite learners to depict relationships—such as plotting variable changes, mapping geometric transformations, or sketching function graphs—promote conceptual understanding through the organization of information in space. The teacher's role is to guide students in interpreting these visuals: identifying what is represented, what correspondences exist between features of the diagram and mathematical entities, and how alternative representations can express the same idea differently. Through this process, learners begin to see how symbolic expressions and verbal explanations can be translated into visual structure and back again.

Visualization also plays a diagnostic role in revealing misconceptions. When a learner's diagram differs from an expected configuration, the discrepancy often exposes a gap in relational understanding. Teachers can use such visual evidence to identify which representational links—verbal, symbolic, or diagrammatic—require reconstruction. Rather than viewing errors as failures, educators can interpret them as indicators of where conceptual rebuilding should occur.

Furthermore, visualization nurtures metacognitive awareness. As students move between diagrams, formulas, and verbal reasoning, they become conscious of their own cognitive strategies and of the representational choices available to them. This awareness supports autonomous learning and the flexible application of knowledge across contexts. In this way, visualization serves not only as a learning aid but also as a foundation for relational competence: the capacity to navigate, connect, and reinterpret ideas through multiple representational systems.

5 Conclusion

This study began with a primary research question: **How do transformations among descriptive, relational, and graphical representations foster relational understanding and support the development of learner agency as envisioned by the OECD Learning Compass 2030?** Through theoretical analysis grounded in Ainsworth's DeFT framework, Duval's semiotic representation theory, and Skemp's distinction between instrumental and relational understanding, this study has demonstrated that representational transformation operates as the fundamental mechanism through which learners construct coherent mathematical understanding. The findings reveal that when learners repeatedly move among the three modes proposed in this study's DRG (Descriptive-Relational-Graphical) model—articulating problems verbally, expressing relationships symbolically, and externalizing structures visually—they develop not merely procedural fluency but relational comprehension that connects meaning across representational systems. This dynamic process of translation and integration cultivates the transformative competencies central to the OECD Learning Compass 2030: autonomy in navigating complex problems, capacity for reflection through metacognitive awareness of representational choices, and agency in constructing knowledge actively rather than receiving it passively.

To address this overarching question, this study pursued three subsidiary inquiries. **First, what are the distinct cognitive functions of descriptive, relational, and graphical representations in mathematical learning, and how do these functions complement one another?** The analysis has shown that the three components of the DRG model serve distinct yet complementary cognitive functions. Descriptive representation enables articulation and reasoning through natural language, providing accessibility and connection to intuitive understanding. Relational representation expresses quantitative and logical structures through symbolic notation, enabling precision and generalization. Graphical representation externalizes spatial and structural relations, supporting perceptual reasoning and pattern recognition. These three modes function complementarily:

descriptive language grounds abstract concepts in communicable meaning, relational structures provide formal rigor, and graphical forms offer intuitive access to relationships that symbolic notation may obscure. Their complementarity lies not in redundancy but in mutual enrichment—each mode reveals aspects of mathematical structure that others cannot fully capture. The DRG model thus provides a comprehensive framework for understanding how these representational systems interact to support mathematical learning.

Second, in what ways does visualization serve as a mediating pathway that enables learners to construct, translate, and integrate meaning across representational systems? This study has demonstrated through geometric examples that visualization—the graphical component of the DRG model—functions not as a secondary illustrative aid but as a cognitive bridge connecting internal cognition with external representation. When learners visualize mathematical relationships—whether through contour maps, geometric diagrams, or function graphs—they reorganize sequential or symbolic information into spatial configurations that reveal structural patterns. This transformation from temporal or symbolic sequence to spatial configuration enables learners to perceive relationships holistically, facilitating both comprehension and error detection. Visualization thus mediates the translation process within the DRG model: it converts verbal descriptions into perceivable forms, renders algebraic relationships spatially interpretable, and provides a shared representational space where descriptive and relational modes can be coordinated and reconciled.

Third, how can the bidirectional and recursive nature of representational transformation be leveraged pedagogically to move learners from instrumental understanding to relational understanding? The pedagogical implications are clear: instruction must design learning environments that actively engage students in constructing, translating, and reflecting upon representations rather than simply receiving them. The bidirectional transformations illustrated in the geometric examples—moving from descriptive to graphical to relational (DG→GR), and from descriptive to relational to graphical (DR→RG)—demonstrate that understanding deepens through recursive cycles of representation and re-representation within the DRG framework. Teachers can leverage this by posing tasks that require students to express the same mathematical idea in multiple forms, to justify why different representations are equivalent, and to choose appropriate representations for particular problem contexts. Such pedagogy shifts the focus from memorizing procedures to reconstructing meaning, thereby fostering the relational understanding that Skemp identified as essential for flexible and transferable knowledge.

By answering these questions through theoretical analysis and examination of geometric examples, this study has demonstrated that representational transformation—as conceptualized through the DRG model—constitutes the core mechanism through which mathematical understanding develops. Learning is revealed not as linear accumulation but as recursive movement through the descriptive, relational, and graphical representational spaces, where meaning emerges from the coordination and integration of multiple expressive systems. Visualization plays a central role in this process, serving as an epistemic medium through which learners externalize internal structures and reorganize conceptual relationships. Ultimately, this paradigm affirms that the capacity to translate, integrate, and visualize knowledge across the DRG framework constitutes the foundation of understanding in mathematics and beyond, supporting the educational vision of learner agency, transformative competence, and well-being articulated in the OECD Learning Compass 2030.

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